Technische Universität Braunschweig

Institut für Theoretische Informatik

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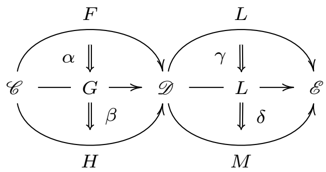
# Algebraic Automata Theory

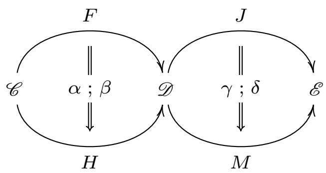
Sheet 1, 2017-11-02

**Exercise 1** [10 Points]

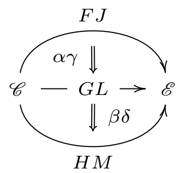
Prove Theorem 1.0.10: ***cat*** (as well as ***Cat*** ) has the middle interchange propert, *i.e.*, in the following situation




the two possible composites agree:




and




**Exercise 2** [12 Points]

(a) Show that every adjunction $F \dashv U$ with $\mathscr{A} \xrightarrow{F} \mathscr{B}$ and $\mathscr{B} \xrightarrow{U} \mathscr{A}$ induces a monad on $\mathscr{A}$ with functor $T = FU$ and the same unit as the unit of the adjunction.

(b) Show that under the hypotheses of (a) there also is a co-monad on $\mathscr{B}$ with functor $S = UF$ and the same co-unit as the co-unit of the adjunction.

(c) Illustrate the above in case of pre-ordered sets $\mathscr{A} = \langle A, \sqsubseteq \rangle$ and $\mathscr{B} = \langle B, \leq \rangle$ , and order-preserving mas $A \xrightarrow{f} B$ resp. $B \xrightarrow{u} A$. Identify the corresponding categories of EM-lgebras resp. EM-co-algebras.

**Exercise 3** [15 Points]

Given a monad $\boldsymbol{T} = \langle T, \eta, \mu \rangle$ on a category $\mathscr{C}$ , the *Kleisli-category* $\mathscr{C}_{\boldsymbol{T}}$ has the same objects as $\mathscr{C}$ , while the hom-sets are given by

$$\langle A, B \rangle \mathscr{C}_{\boldsymbol{T}} = \langle A, BT \rangle \mathscr{C}$$

The composition of the $\mathscr{C}$-morphisms $A \xrightarrow{f} BT$ and $B \xrightarrow{g} CT$ in $\mathscr{C}_{\boldsymbol{T}}$ is defined by

$$\langle A \xrightarrow{f} BT, B \xrightarrow{g} CT \rangle \mapsto (A \xrightarrow{f} BT \xrightarrow{gT} CTT \xrightarrow{C\mu} CT$$

(a) Compare $\mathscr{C}_{\boldsymbol{T}}$ with the full subcategory of $\mathscr{C}^{\boldsymbol{T}}$ spanned by the free algebras over $\mathscr{C}$-objects.

(b) Identify the Kleisli-category for the power-set monad.

**due on Thursday, 2017-11-09, 13:15,**