Technische Universität Braunschweig

Institut für Theoretische Informatik

Dr. Jürgen Koslowski

Algebraic Automata Theory

Sheet 5, 2017-11-23

Exercise 1 [16 Points]

An *ideal* of a monoid $\langle M, \cdot, e\rangle$ is a subset $A \subseteq M$ that satisfies $M \cdot A \subseteq A \supseteq A \cdot M$.

1. Show that the set $\mathcal{M}$-***idl*** of all ideals of M is a submonoid of the power-set $M\mathbb{P}$.
2. Analize the closure properties of M***idl*** wrt. the Boolean operations union, intersection, and complement.
3. How do ideals behave wrt. residuation?
4. How do ideals behave wrt. direct and inverse images?

Exercise 2 [14 Points]

Monoids with an absorbing element 0 or $\perp$ are also known as *binoids*. Their homomorphisms have to preserve the absorbing element.

1. Show that a monoid can have at most one absorbing element.
2. Check, whether or not the category ***bin*** of binoids and their homomorphisms is a full subcategory of ***mon*** .
3. Try to analyze the notion of ideal in ***bin*** .

due on Thursday, 2017-11-30, 13:15,