Applied Automata Theory (SS 2011) Out: Thu, July 14 Due: Wed, July 21

Exercise Sheet 11

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Let A be a bottom-up tree automaton and q be a state in A. In the following exercises, by an *A-tree for q*, we mean a tree that is labeled with states by a run of A and in which the root node is labeled with q.

Exercise 11.1

Consider the algorithm for deciding emptiness of a regular tree language. Let A be a bottom-up tree automaton.

(a) Show that for $q \in R_k \setminus R_{k-1}$ there is an A-tree for q, and that k is the smallest height for which such a tree exists.

(b) Prove that $L(A) \neq \emptyset$ implies $R \cap Q_F \neq \emptyset$. To this end, show by induction on k that if there is an A-tree for q of height $\leq k$, then $q \in R_k$.

(c) Apply the algorithm to the bottom-up tree automaton $A = (\{q_0, q_1, q_2\}, \rightarrow, \{q_2\})$ with

$$(q_i, q_j) \longrightarrow_a q_{\min(i,j)} \qquad (q_i, q_j) \longrightarrow_b q_{\min(\min(i,j)+1,2)} \qquad \longrightarrow_c q_0$$

for $i, j \in \{0, 1, 2\}$.

Exercise 11.2

Present a bottom-up tree automaton (with $\Box$) that recognizes (up to α-conversion) the language of solutions of $x(\lambda y_1 \lambda y_2. f(a, y_2)) = f(a, g(a))$.

Exercise 11.3

Show that the class of regular tree languages is closed under intersection.

Exercise 11.4

Show that it is decidable whether the tree language accepted by a given bottom-up tree automaton is finite. *Hint:* In a first step, use the construction from the emptiness check to eliminate unproductive states. Here, a state q is called productive if there is an A-tree for q. Then search for loops.