|  | Advanced Automata Theory |  |
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| Prof Roland Meyer | Exercise Sheet 7 | TU Braunschweig |
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## Exercise 1: NBA Complementation

Consider the NBA $A$ over $\Sigma=\{a, b\}$ below:


Use Büchi's complementation method discussed in class to compute $L(A)$ and $\overline{L(A)}$.

## Exercise 2: Equivalence

Consider an NBA $A$, two classes $[u]_{\sim_{A}}$ and $[v]_{\sim_{A}}$ of $\sim_{A}$, and $w \in[u]_{\sim_{A}} \cdot[v]_{\sim_{A}}^{\omega}$ an $\omega$-word. Show that if $w \in L(A)$ then $[u]_{\sim A} \cdot[v]_{\sim A}^{\omega} \subseteq L(A)$.

## Exercise 3: Muller Automata

A Nondeterministic Muller Automaton (NMA) is a tuple $A=\left(Q, \Sigma, \delta, q_{0}, F\right)$. The first four components are as in Büchi automata. $F=\left\{Q_{F}^{1}, \ldots, Q_{F}^{n}\right\} \subseteq \mathcal{P}(Q)$ is a set of sets of states instead of a single set of states. The idea is to accept a run if the set of states that occur infinitely often matches one of the $Q_{F}^{i}$ exactly. Formally, a run $r$ of $A$ is accepting if $\operatorname{Inf}(r) \in F$ where $\operatorname{Inf}(r)$ is the set of states that are visited infinitely often in $r$. As for Büchi automata, we call $A$ a Deterministic Muller Automaton (DMA) if for each $q \in Q$ and $a \in \Sigma$ there is exactly one state $q^{\prime} \in Q$ such that $\left(q, a, q^{\prime}\right) \in \delta$.
a) Given an NBA $A$, show that there is an NMA $A_{N M A}$ such that $L\left(A_{N M A}\right)=L(A)$.
b) Show that DMA are strictly more expressive than DBA.
c) Given a DMA $A$, show that there is an NBA $A_{N B A}$ such that $L\left(A_{N B A}\right)=L(A)$.
d) Prove that DMA are closed under complement, i.e. for every DMA $A$ there exists a DMA $\bar{A}$ with $L(\bar{A})=\overline{L(A)}$.

