Advanced Automata Theory

Prof Roland Meyer

Exercise Sheet 12

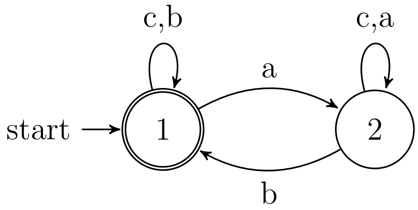
TU Braunschweig
Dr Prakash Saivasan

Summer term 2017

Out: July 5 Due: July 10, 12:00

Exercise 1: Transition Monoids

Given a finite automaton $A = (Q, \Sigma, \delta, s, F)$, let $M_A = (\{\hat{\delta}_x \mid x \in \Sigma^*\}, \circ, \hat{\delta}_\epsilon = Id_Q)$ where, $\hat{\delta}_x : Q \mapsto Q$ is given by $\hat{\delta}_x(q) = \delta(q, x)$, $\circ$ is the composition operator (for functions f, g, $f \circ g(x) = g(f(x))$) and Id_Q is the identity function. Such a monoid M_A is called the transition monoid of the given automata A. Consider the following automata and construct the transition monoid for the same.



Exercise 2: Idempotent

Prove that for every finite monoid $S = (M, \cdot, 1)$, for any $a \in M$, there are $n, p \in \mathbb{N}$ such that $a^n = a^{n+p}$.

Exercise 3: Unique zero element

Prove that for every finite monoid, if there is a zero element, it is unique. We say an element $s \in M$ is a zero element if $SsS = \{s\}$