SS 2018 02.04.2018

Exercises to the lecture
Algorithmic Automata Theory
Sheet 3

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Peter Chini Delivery until 08.05.2018 at 12:00

Exercise 3.1 (Extensions of WMSO)

a) Let us consider WMSO$[<, \text{suc}, .\,]$, the set of WMSO-formulas extended by concatenation. This means if φ, ψ are WMSO$[<, \text{suc}, .\,]$-formulas, then $\varphi.\psi$ is also a WMSO$[<, \text{suc}, .\,]$-formula. Give semantics to concatenation (i.e. define when $\mathcal{S}(w), \mathcal{I} \models \varphi.\psi$ should be satisfied) so that

$$L(\varphi.\psi) = L(\varphi).L(\psi).$$

b) Present a WMSO$[<, \text{suc}]$ formula that is equivalent to $\varphi.\psi$.

c) For some fixed alphabet Σ, let us consider WMSO$[<, \text{suc}, [a]_{a\in\Sigma}]$, the set of WMSO-formulas extended by an operator $[a]$ for each symbol of the alphabet. If φ is a WMSO$[<, \text{suc}, [a]_{a\in\Sigma}]$-formula, then $[a]\varphi$ for any $a \in \Sigma$ is a WMSO$[<, \text{suc}, [a]_{a\in\Sigma}]$-formula as well. Give semantics to $[a]\varphi$ so that

$$L([a]\varphi) = \{w \in \Sigma^* \mid aw \in L(\varphi)\}.$$

Exercise 3.2 (Ehrenfeucht-Fraïssé Games)
Let $n \in \mathbb{N}$ be arbitrary. Which is the maximal number of rounds $k \in \mathbb{N}$ such that the duplicator has a winning strategy for $G_k\left((ab)^{2n+1}, (ba)^{2n+1}\right)$?
Hint: First see what happens for $n = 1$ and $n = 2$.

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