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Exercises to the lecture
Algorithmic Automata Theory
Sheet 4

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Peter Chini Delivery until 15.05.2018 at 12:00

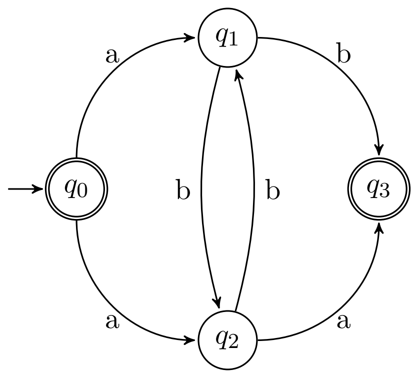
Exercise 4.1 (Finite Monoids)
Let M be a finite monoid. Prove the existence of an idempotent element in M: An element $t \in M$ such that $t \cdot t = t$.

Hint: Take an element $s \in M$ and consider the sequence $(s_i)_{i \in \mathbb{N}}$. Show that there are $i, p \in \mathbb{N}, p > 0$ such that $s^{m+p} = s^m$ for any $m \geq i$.

Exercise 4.2 (Transition Monoid)
Let $A = (\Sigma, Q, q_0, \delta, Q_F)$ be an NFA. Note that the transition relation can be seen as a function $\delta : Q \times \Sigma \to \mathcal{P}(Q)$. We define the *transition monoid* to be the set

$$M = \{\rho_x \mid x \in \Sigma^*, \rho_x(q) = \delta(q, x) \text{ for all } q \in Q\}.$$

Consider the automaton given below. Determine its transition monoid.



Exercise 4.3 (Equivalence Classes)
Let $M = \{f : \{1, 2, 3\} \to \{1, 2, 3\} \mid f \text{ a function}\}$ be the set of all function from $\{1, 2, 3\}$ to $\{1, 2, 3\}$. Then, $(M, \circ, \mathrm{id})$ is a finite monoid. Note that $\circ$ is concatenation from the left. In $f \circ g$, f is applied first, g afterwards: $(f \circ g)(x) = g(f(x))$. For functions in M, we use shortcut notation: $[i\, j\, k]$ denotes the function mapping 1 to i, 2 to j, and 3 to k.

a) Find a closed form for the class $R([1\,2\,1])$.

b) Find a closed form for the class $L([1\,2\,1])$.

c) Find a closed form for the class $H([1\,2\,1])$.

Hint: Recall that $R(f) = f \circ M$ and $L(f) = M \circ f$.

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