Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 3

Jun.-Prof. Roland Meyer, Reiner Hüchting, Georgel Călin Due: Tue, Nov 6 **(noon)**

Exercise 3.1

Suppose the logic WMSO[$<$, suc, $\sqsubseteq$] is obtained from WMSO[$<$, suc] by adding the predicate $\sqsubseteq$, which is interpreted as

$$S_w, I \models X \sqsubseteq Y \quad :\Longleftrightarrow \quad \text{for each } x \in I(X)\text{, there is a } y \in I(Y) \text{ such that } x \leq y.$$

Extend the proof of Theorem Büchi II to WMSO[$<$, suc, $\sqsubseteq$].

Exercise 3.2

Similar to $\exists$WMSO, we define *universal* WMSO, denoted by $\forall$WMSO, as the syntactic restriction of WMSO to formulas

$$\forall X_1 : \ldots \forall X_n : \varphi$$

where φ does not contain second-order quantification.

Show that a language is regular iff it is $\forall$WMSO-definable.

Exercise 3.3

In the lecture, a winning strategy for Duplicator was presented for $G_k(a^i, a^j)$, provided that $i \geq 2^k - 1$ and $j \geq 2^k - 1$.

(a) Show, again by induction on k, that for any $u, v, w \in \Sigma^*$, there is a winning strategy for Duplicator for $G_k(uv^iw, uv^jw)$, if $i \geq 2^k - 1$ and $j \geq 2^k - 1$. (Your proof need not be more detailed than the one in the lecture).

(b) Explain why there is no winning strategy for Duplicator when $i, j < 2^k - 1$ and $i \neq j$. That is, *roughly* describe a winning strategy for Spoiler (neither a formal proof nor exact calculations are necessary here).

Exercise 3.4

Consider languages $(L_n)_{n\geq 2}$ with $L_n = (a+b)^*a(a+b)^{n-1}$ over $\Sigma = \{a, b\}$. Give an NFA A_n with $n+1$ states that accepts L_n and prove no DFA with less than 2^n states accepts it. *Hint: Think of words of length n over Σ.*