Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 12

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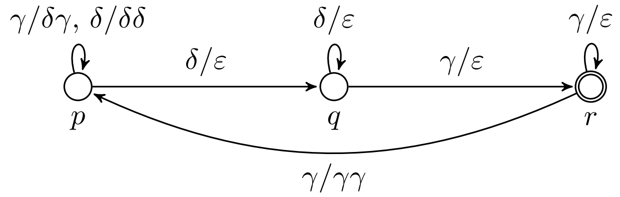
Exercise 12.1 Büchi Pushdown Systems

Determine the set of configurations from which an infinite accepting run is possible in the following Büchi-pushdown system:



Exercise 12.2 Non-erasing Context Free Grammars

Let Σ be ranked alphabet. A context-free grammar is *non-erasing* if the empty word does not occur on the right side of any production.

(a) Show that for each regular tree language L, there is a non-erasing context-free grammar G with $\text{yield}(L) = \mathsf{L}(G)$.

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Exercise 12.3 BUTA Language Emptiness

Consider the algorithm for deciding emptiness of a regular tree language accepted by a bottom-up tree automaton A.

(a) Show that for $q \in R_k \setminus R_{k-1}$ there is an A-tree for q [1], and that k is the smallest height for which such a tree exists.

(b) Prove that $L(A) \neq \emptyset$ implies $R \cap Q_F \neq \emptyset$. To this end, show by induction on k that if there is an A-tree for q of height $\leq k$, then $q \in R_k$.

(c) Apply the algorithm to bottom-up tree automaton $A = (\{q_0, q_1, q_2\}, \rightarrow, \{q_2\})$ with

$$(q_i, q_j) \longrightarrow_a q_{\min(i,j)} \qquad (q_i, q_j) \longrightarrow_b q_{\min(\min(i,j)+1,2)} \qquad \longrightarrow_c q_0$$

for $i, j \in \{0, 1, 2\}$.

[1] A-tree for q means a tree labeled with states by a run of A and with the root node labeled by q.

Exercise 12.4 Finiteness of BUTA Tree Language

Show that it is decidable whether the tree language accepted by a given bottom-up tree automaton is finite.

Hint: use the construction for the emptiness check to eliminate unproductive states and then search for loops. We call a state q productive if there is an A-tree for q.