Applied Automata Theory (WS 2012/2013) Technische Universität Kaiserslautern

Exercise Sheet 14

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Exercise 14.1 Finite Words

(a) Consider a deck of cards (with **arbitrary many** cards) in which black and red cards alternate, and the top card is black. Cut the deck at any point into two piles, and then perform a riffle (also called a dovetail shuffle) to yield a new deck.

E.g., we can cut the deck with six cards 123456 into two piles 12 and 3456, and the riffle yields 132456 or 312456, depending on the pile we start the riffle with. Now, take the cards from the new deck two at a time (if the riffle yields 132456, then this exposes cards 3, 4, and 6; if it yields 314256, it exposes cards 1, 2, and 6).

Prove with help of regular expressions that all exposed cards have the same color.

Hint: the expression $(BR)^*(\epsilon + B)$ represents the possible initial decks

(b) Let L be an arbitrary language over a 1-letter alphabet. Prove that L^* is regular.

(c) Consider the following system of equations, where the variables X, Y represent languages over the alphabet $\Sigma = \{a, b, c, d, e, f\}$:

$$X = \{a\}X \cup \{b\}Y \cup \{c\}$$
$$Y = \{d\}X \cup \{e\}Y \cup \{f\}$$

Find the unique minimal solution of the system with the help of Arden's lemma.

(d) Let $L, U, V \subseteq \Sigma^*$ such that $\epsilon \in U$. Prove that the L-solutions of $L = UL \cup V$ are precisely the languages in the set $\{U^*V' \mid V \subseteq V' \subseteq \Sigma^*\}$.

(e) Give a defining WMSO-formula, an automaton, and a regular expression for the following languages over $\{a, b\}$:

- the set of words of even length and containing only a's or only b's.
- the set of words, where between each two b's with no other b inbetween there is a block of an odd number of letters a.
- the set of words with an odd length and an odd number of occurrences of a.

(f) For which $k \in \mathbb{N}$ does the Duplicator win $G_k(u, v)$ for the following words u,v:

- $u = aabaabaaba$ and $v = abaabaabaa$
- $u = a^n b a^{n+1}$ and $v = a^{n+1} b a^n$ for $n \in \mathbb{N}$.

[0]this exercise sheet is not compulsory – most exercises are from the literature cited on the course page

(g) Compute the Parikh image $\Psi(\mathsf{L}(G))$ for the following context free grammar G:

$$S \to aSb \mid ab \mid S_1S' \qquad S' \to SS_2 \qquad S_1 \to a \qquad S_2 \to b$$

(h) Give a language L such that its Parikh image $\Psi(L)$ is not semilinear.

Exercise 14.2 Infinite Words

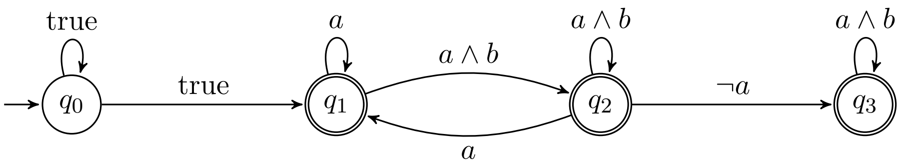
(a) Given k processes and automata $A_1, \ldots, A_k$ representing them, present a fair scheduler enforcing that process 1 is active for 10 CPU cycles at least every 100 cycles. Keep the scheduler as general as possible.

(b) Consider the following statement: *Let A be an NFA such that $L(A) = L$ and $\epsilon \notin L$. If we see A as an NBA, then $L(A) = L^\omega$.* Find a counterexample for the statement.

(c) Let $a = \emptyset$, $b = \{p\}$, $c = \{q\}$, and $d = \{p, q\}$. Give LTL formulas to define the following languages over $\Sigma = 2^{\{p,q\}}$:

- $a^*b^*c^*\Sigma^\omega$
- $(a^+b^+c^+d^+)^\omega$
- $\{w \in \Sigma^\omega \mid |w|_a = \infty \text{ implies } |w|_b = \infty\}$
- $(\Sigma^*a\Sigma^*b\Sigma^*c)^\omega$
- $\{q \in \Sigma^\omega \mid \text{between any } d\text{'s in } w \text{ there is at least a } c\}$

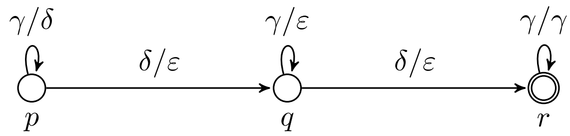
(d) Consider the GNBA G below over $\mathcal{P} = \{a, b\}$ with $Q_F^1 = \{q_1, q_3\}$ and $Q_F^2 = \{q_2\}$



- provide an LTL formula φ such that $L(\varphi) = L(G)$
- depict the NBA A_φ such that $L(A_\varphi) = L(G)$.

(e) Use the Vardi-Wolper construction to contruct the GNBA for $(a \wedge \bigcirc a)\mathcal{U}\neg a$.

(f) Determine the set of configurations from which an infinite accepting run is possible in the following Büchi-pushdown system:



Exercise 14.3 Finite Trees and Parity Games

(a) Which of the following tree languages are accepted by some finite tree automaton over $\Sigma = \{a, b, c, d\}$ with $rk(a) = rk(b) = 2$ and $rk(c) = rk(d) = 0$?

- $L_1 := \{t \in \mathcal{T}_\Sigma \mid \text{the path } \epsilon, 0, 01, 010, 0101, \ldots \text{ in } t \text{ contains an even number of } a\text{'s}\}$
- $L_2 := \{t \in \mathcal{T}_\Sigma \mid t \text{ is an unbalanced tree}\}$
- $L_3 := \{t \in \mathcal{T}_\Sigma \mid \text{there are nodes } u, v \text{ in } t \text{ with } t(u) = c, t(v) = d \text{ and } u \text{ is left of } v\}$
- $L_4 := \{t \in \mathcal{T}_\Sigma \mid t \text{ has exactly 239 leaves, all of which are labelled by } c\}$

(b) Let $A = (Q, \Sigma, Q_F, \rightarrow)$ be a **normalized** NHA. A is normalized if there is always a single rule regexp $\rightarrow_a q$ for every $a \in \Sigma$ and $q \in Q$. We further call A **top-down deterministic**, i.e. top-down DHA, if $|Q_F| = 1$ and for all transitions regexp $\rightarrow_a q$ there is for each n at most one word of length n in regexp.

Give a recognizable language $L \subseteq \mathcal{T}_\Sigma$ that cannot be recognized by a top-down DHA.

(c) Determine the winning regions of A and P for the following Parity games: