Applied Automata Theory (WS 2013/2014) Technische Universität Kaiserslautern

Exercise Sheet 5

Jun.-Prof. Roland Meyer, Georgel Calin Due: Tue, Nov 26

Exercise 5.1 Semilinear Sets

Let $S = \bigcup_{i \in \{1,\ldots,l\}} \mathcal{L}(c_i, P_i) \subseteq \mathbb{N}^n$ be semi-linear. Prove closure under Kleene iteration:

$$\{v_1 + \ldots + v_k \mid k \in \mathbb{N} \text{ and } v_1, \ldots, v_k \in S\} = \bigcup_{I \subseteq \{1,\ldots,l\}} \mathcal{L}\big(\sum_{i \in I} c_i, \bigcup_{i \in I} P_i \cup \{c_i\}\big).$$

Exercise 5.2 Parikh Images of Regular Languages **not graded**

(a) Prove that $\Psi(L)$ is semilinear if $L \in \mathrm{REG}_\Sigma$.

(b) Prove that for a semilinear set $S \subseteq \mathbb{N}^n$ there is a regular language L with $S = \Psi(L)$.

Exercise 5.3 Parikh Images of Context Free Languages

Use the method from class to compute $\Psi(\mathsf{L}(G))$ for the grammar G whose rules are:

(a) $S \to ab \mid S_1 S'$, $S' \to S S_2$, $S_1 \to a$, $S_2 \to b$

(b) $S \to S_1 S_2 \mid \varepsilon$, $S_1 \to aSb$, $S_2 \to bSc$

Exercise 5.4 Presburger Extensions of Context Free Languages

We define extended context-free grammars (G, φ) where G is a context-free grammar over alphabet Σ and φ is a Presburger formula with free variables x_a for all $a \in \Sigma$ such that $L(G, \varphi) := \{w \in \Sigma^* \mid w \in L(G) \text{ and } \Psi(w) \vDash \varphi\}$.

(a) Prove that emptiness of $L(G, \varphi)$ is NP-hard for any extended CFG (G, φ) and that when φ is an existential Presburger formula it is also in NP.

Hint: satisfiability of existential Presburger formulas is known to be NP-complete.

(b) Find an extended CFG (G, φ) such that $L(G) \notin REG_\Sigma$ and $a^n b^n c^n = L(G, \varphi)$.