**Applied Automata Theory** (WS 2013/2014) Technische Universität Kaiserslautern

# Exercise Sheet 6

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## Exercise 6.1 Marking Equation

Let $N = (S, T, W)$ be a Petri net with connectivity matrix $\mathbb{C}$, $M_1, M_2 \in \mathbb{N}^{|S|}$, and $\sigma \in T^*$ such that $M_1 \xrightarrow{\sigma} M_2$. Prove by induction on the length of $\sigma$ that $M_2 = M_1 + \mathbb{C} \cdot \Psi(\sigma)$.

*Hint:* $\mathbb{C}(\bullet, t) = \mathbb{C} \cdot E_t$, *where* $E_t$ *is the unit vector with* 1 *at position* $t$ *and* 0 *elsewhere.*

## Exercise 6.2 Communication-free Petri Nets of Context-free Languages

Compute the communication-free Petri nets of the following context-free grammars and find the smallest $X$ for which (a) and (b) in Esparza's theorem from class hold:

(a) $S \to aSbS' \mid \varepsilon$, $S' \to SbS'a \mid \varepsilon$

(b) $S \to S' \mid \varepsilon$, $S' \to aSb \mid bSc$

*Hint:* $N \to \varepsilon$ *productions are encoded by transitions with no outgoing arcs.*

## Exercise 6.3 Existential Presburger Formulas for Context-free Languages

Use the method from class to compute existential Presburger formulas $\varphi_G$ such that

(a) $Sol(\varphi_G) = \Psi(L(G))$ for grammar $G$ described by the productions in 6.2.(a), and

(b) $Sol(\varphi_G) = \Psi(L(G))$ for grammar $G$ described by the productions in 6.2.(b).

*Hint: use the Petri nets you built in 6.2 and clearly name your variables.*

## Exercise 6.4 Linear-time Construction of Existential Presburger Formula

Turn the method into a linear time (pseudocode) algorithm that omits the Petri net computation.

More precisely, upon linearly processing the production rules of $G$, your algorithm should output a linear-size existential Presburger formula $\varphi_G$ with $Sol(\varphi_G) = \Psi(L(G))$.