

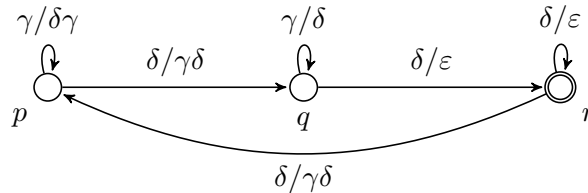
## Exercise Sheet 10

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Due: Tue, Jan 14

### Exercise 10.1 Büchi Pushdown Systems

Solve the accepting run problem for the Büchi-pushdown system over  $\Gamma = \{\gamma, \delta\}$  below:



- (a) Find all  $(s, \gamma) \in Q \times \Gamma$  such that  $(s, \gamma) \rightarrow^+ (r, u) \rightarrow^* (s, \gamma \cdot v)$  for some  $u, v \in \Gamma^*$ .
- (b) Compute  $A_{\text{pre}^*(C)}$  for  $C = \{(s, \gamma \cdot \Gamma^*) \mid (s, \gamma) \text{ is a configuration found in (a)}\}$ .

*Note: the lowercase  $\gamma \in \Gamma$  in (a) and (b) is an arbitrary symbol.*

### Exercise 10.2 Boolean Satisfiability

Let  $\Sigma = \{\wedge, \vee, \neg, 0, 1, x\}$  with  $rk(\wedge) = rk(\vee) = 2$ ,  $rk(\neg) = 1$ ,  $rk(0) = rk(1) = rk(x) = 0$ .

- (a) Give a deterministic BUTA that recognises the satisfiable Boolean formulas over  $x$ .
- (b) Use (a) to establish whether  $\neg(x \wedge 0) \vee (\neg x \wedge 1)$  and  $(x \vee 0) \wedge (\neg x \wedge 1)$  are satisfiable.

### Exercise 10.3 Tree Language Acceptance

Let  $\Sigma = \{a, b, c, d\}$  with  $rk(a) = rk(b) = 2$ , respectively  $rk(c) = rk(d) = 0$ . Establish which of the following tree languages are accepted by some BUTA.

- (a)  $L_1 := \{t \in \mathcal{T}_\Sigma \mid \text{the path } \epsilon, 0, 01, 010, 0101, \dots \text{ in } t \text{ contains an even number of } a\text{'s}\}$ .
- (b)  $L_2 := \{t \in \mathcal{T}_\Sigma \mid t \text{ is an unbalanced tree}\}$ .
- (c)  $L_3 := \{t \in \mathcal{T}_\Sigma \mid \text{there are nodes } u, v \text{ in } t \text{ with } t(u) = c, t(v) = d \text{ and } u \text{ is left of } v\}$ .
- (d)  $L_4 := \{t \in \mathcal{T}_\Sigma \mid \text{precisely 2014 of } t\text{'s leaves are labelled by } c\}$ .

Determine which of the languages above are also accepted by a deterministic TDTA.

### Exercise 10.4 Emptiness of TDTA

Give an algorithm that decides emptiness of nondeterministic TDTA without checking emptiness of an equivalently encoded BUTA.

Prove termination and correctness of your algorithm.