Applied Automata Theory (WS 2014/2015) Technische Universität Kaiserslautern

Exercise Sheet 2

Jun.-Prof. Roland Meyer, Reiner Hüchting Due: Tue, 11 November

Exercise 2.1 $\mathrm{WMSO}[<,\mathrm{suc}]$-defined Languages

(a) Present a $\mathrm{WMSO}[<,\mathrm{suc}]$-formula that defines the language $b^*a^+b(a+b)^*$.

(b) Present a $\mathrm{WMSO}[<,\mathrm{suc}]$-formula that defines the language $((aa)^*b)^*$.

(c) Present a $\mathrm{WMSO}[<,\mathrm{suc}]$-formula that defines all finite words over $\Sigma = \{a_0, \dots, a_{n-1}\}$ such that every letter a_i is always immediately followed by $a_{i+1 \mod n}$ for $0 \leq i < n$.

(d) What is the language described by $\exists y\, \forall x\, \forall z.\, x < y \wedge y < z \rightarrow \neg P_a(x) \wedge P_b(y)$?

Exercise 2.2 Weak Dyadic Second Order Logic

Let WDSO be like WMSO with the modification that all second order variables X are dyadic instead of being monadic, i.e. one has atomic formulas $X(x, y)$. The syntax and semantics of WDSO are the same with those of WMSO up to the predicate replacement:

$S(w), I \vDash X(x, y)$ iff. $(I(x), I(y)) \in I(X)$

$S(w), I \vDash \exists X.\varphi$ iff. there is a finite set $M \subseteq D(w)^2$ such that $I[M/X] \vDash \varphi$.

Give (with arguments) a WDSO-formula that defines the language $\{a^n b^n \mid n \geq 0\}$.

Exercise 2.3 From WMSO to Finite Automata

Using the method presented in the lecture, construct a finite automaton that accepts the language defined by the formula $\varphi = \exists x :\ P_a(x) \wedge \forall y :\ x < y \rightarrow P_b(y)$.

Exercise 2.4 Ehrenfeucht-Fraïssé Games

Let $n \in \mathbb{N}$ be arbitrarily fixed. Which is the maximal number of rounds $k \in \mathbb{N}$ such that the Duplicator has a winning strategy for $G_k((ab)^{2n+1}, (ba)^{2n+1})$?

Hint: first see what happens when n=1 and n=2.