Applied Automata Theory (WS 2014/2015) Technische Universität Kaiserslautern

Exercise Sheet 3

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Exercise 3.1 Extending WMSO

Suppose the logic WMSO$[<, \text{suc}, \sqsubseteq]$ is obtained from WMSO$[<, \text{suc}]$ by adding the predicate $\sqsubseteq$, which is interpreted as

$$S_w, I \models X \sqsubseteq Y \quad :\Longleftrightarrow \quad \text{for each } x \in I(X), \text{ there is a } y \in I(Y) \text{ such that } x \leq y.$$

Extend the proof of Theorem Büchi II to WMSO$[<, \text{suc}, \sqsubseteq]$.

Exercise 3.2 WMSO Model Checking

Given an automaton A and a WMSO$[<, \text{suc}]$-formula φ, the *model checking* problem asks whether every word accepted by A satisfies φ. If yes, we write $A \models \varphi$.

(a) Show that the model checking problem is Turing reducible to the emptiness problem for finite automata

(b) Show that the emptiness problem is Turing reducible to the problem of whether in a finite automaton, one given state can be reached.

Exercise 3.3 Star-Free Languages

Prove or disprove whether the following languages are star-free:

(a) $(ab + ba)^*$

(b) $(abab)^*$

(c) $(a + bab)^*$

Exercise 3.4 Star-Free $\Rightarrow$ FO$[<]$-definable

(a) Let $w = a_0 \dots a_n \in \Sigma^*$ and let $i, j \in \mathbb{N}$ such that $0 \leq i \leq j \leq n$. Show that for every FO$[<]$-sentence φ and FO-variables x, y with $I(x) = i, I(y) = j$, there is a formula $\psi(x, y)$ such that

$$S(w), I \vDash \psi \text{ if and only if } S(a_i \dots a_j) \vDash \varphi.$$

(b) Deduce from (a) that FO$[<]$-definable languages are closed under concatenation.

(c) Conclude by structural induction that every star-free language is FO$[<]$-definable.