

Exercise Sheet 6

Prof. Dr. Roland Meyer, Reiner Hüchting

Due: Tue, Dec 9

Exercise 6.1 Marking Equation

Let $N = (S, T, W)$ be a Petri net with connectivity matrix \mathbb{C} , $M_1, M_2 \in \mathbb{N}^{|S|}$, and $\sigma \in T^*$ such that $M_1 \xrightarrow{\sigma} M_2$. Prove by induction on the length of σ that $M_2 = M_1 + \mathbb{C} \cdot \Psi(\sigma)$.

Hint: $\mathbb{C}(\bullet, t) = \mathbb{C} \cdot E_t$, where E_t is the unit vector with 1 at position t and 0 elsewhere.

Exercise 6.2 Communication-free Petri Nets of Context-free Languages

Compute the communication-free Petri nets of the following context-free grammars and find the smallest X for which (a) and (b) in Esparza's theorem from class hold:

(a) $S \rightarrow aSbS' \mid \varepsilon, S' \rightarrow SbS'a \mid \varepsilon$

(b) $S \rightarrow S' \mid \varepsilon, S' \rightarrow aSb \mid bSc$

Hint: $N \rightarrow \varepsilon$ productions are encoded by transitions with no outgoing arcs.

Exercise 6.3 Existential Presburger Formulas for Context-free Languages

Use the method from class to compute existential Presburger formulas φ_G such that

(a) $Sol(\varphi_G) = \Psi(L(G))$ for grammar G described by the productions in 6.2.(a), and

(b) $Sol(\varphi_G) = \Psi(L(G))$ for grammar G described by the productions in 6.2.(b).

Hint: use the Petri nets you built in 6.2 and clearly name your variables.

Exercise 6.4 Linear-time Construction of Existential Presburger Formula

Turn the method into a linear time (pseudocode) algorithm that omits the Petri net computation.

More precisely, upon linearly processing the production rules of G , your algorithm should output a linear-size existential Presburger formula φ_G with $Sol(\varphi_G) = \Psi(L(G))$.