Applied Automata Theory (WS 2014/2015) Technische Universität Kaiserslautern

Exercise Sheet 8

Prof. Dr. Roland Meyer, Reiner Hüchting Due: Tue, Jan 06

Exercise 8.1 Verifying Operating Systems

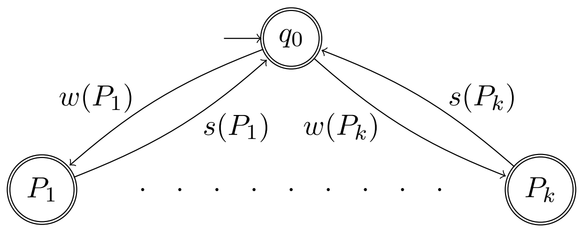
Our goal is to verify an operating system that runs k processes and has a scheduler. Consider the following Büchi automata:

$A_{\text{OS}} := A_{P_1} \parallel \ldots \parallel A_{P_k}$:	Describes the behaviour of the operating system, where A_{P_i} represents the behavior of process P_i.
A_{Sched}:	Describes the scheduling strategy.
A_{Prop}:	Describes a property to be checked.

Our verification task amounts to solving the following model checking problem:

$$\mathsf{L}(A_{\text{OS}}) \cap \mathsf{L}(A_{\text{Sched}}) \subseteq \mathsf{L}(A_{\text{Prop}}).$$

To solve this problem in a general way, we introduce a *most general* scheduling Büchi automaton A_{MG} that allows for arbitrary behaviours of the scheduler:



The scheduler can randomly *wake up* ($w(P_i)$) and *suspend* ($s(P_i)$) processes and the processes only work when awake. Unfortunately, this general scheduler is not *fair*: it does not necessarily wake up each process infinitely often.

(a) Modify A_{MG} to a *fair* automaton A_{MGF} that wakes up every process infinitely often. Keep A_{MGF} as general as possible, do not implement a concrete scheduling strategy.

(b) Present an automaton A_{RR} that describes the *Round Robin* scheduling strategy. What is the relationship between $\mathsf{L}(A_{\text{RR}})$ and $\mathsf{L}(A_{\text{MG}})$ respectively $\mathsf{L}(A_{MGF})$?

(c) Why can you conclude $\mathsf{L}(A_{\text{OS}}) \cap \mathsf{L}(A_{\text{RR}}) \subseteq \mathsf{L}(A_{\text{Prop}})$ from $\mathsf{L}(A_{\text{OS}}) \cap \mathsf{L}(A_{\text{MGF}}) \subseteq \mathsf{L}(A_{\text{Prop}})$?

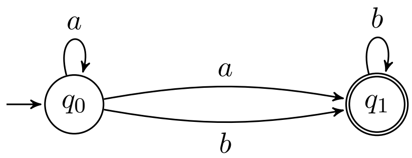
Exercise 8.2 NBA Emptiness and Membership

Let A be an NBA and uv^ω be an ω-word. Give algorithms that decide whether:

$$L(A) = \emptyset \qquad\qquad uv^\omega \in L(A).$$

Exercise 8.3 NBA Complementation

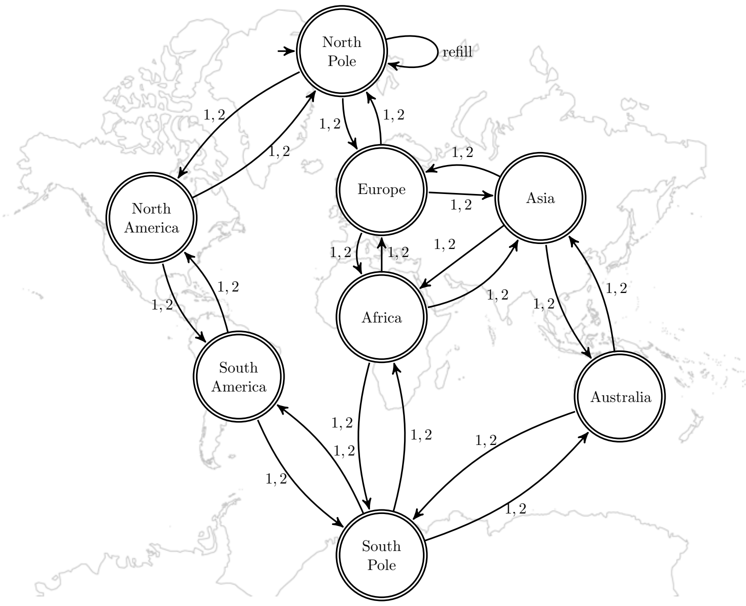
Consider the NBA A over $\Sigma = \{a, b\}$ below:



Use Büchi's complementation method discussed in class to compute $L(A)$ and $\overline{L(A)}$.

Exercise 8.4 Travelling Santa

Santa has decided to swap his traditional sleigh for the brand new *Chrismas Racer 3000*, equipped with the state of the art automata driven navigation system *Rudolph Go v0.99beta*. The automaton $A_{Rudolph}$ controlling *Rudolph* is depicted below:



Each time Santa lifts off, *Rudolph* randomly chooses a neighbouring continent. Santa drops one or two presents every time he enters a region and he can refill at the north pole. Currently, *Rudolph* cannot prevent Santa from running out of presents. Please help Santa by upgrading *Rudolph* to version 1.0:

(a) Determine the minimum present capacity of the sleigh needed not to run empty.

(b) Give an NBA A_{Sleigh} modelling how the number of presents in Santa's sleigh changes.

(c) Explain how one can use $A_{Rudolph}$ and A_{Sleigh} to create a controller that guarantees Santa to never run out of presents.

(d) How can you modify the controller so that all continents are visited infinitely often?