Exercises to the lecture
Complexity Theory
Sheet 3

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Exercise 3.1 (Intersection Non-Emptiness of Regular Languages)

Consider the following problem.

Intersection Non-Emptiness of Regular Langauges (INE)	
Input:	Non-deterministic finite automata $A_1, \ldots, A_k$ for a $k \in \mathbb{N}$.
Question:	$\bigcap_{i=1}^{k} L(A_i) \neq \emptyset$?

Show that INE is PSPACE-complete.

Hint: For the hardness, reduce from the reachability problem for safe Petri Nets. Note that an execution of a Petri Net is a sequence of firings. Firing a transition just amounts to putting and consuming tokens. Construct automata over the alphabet $\{put_p\,, consume_p \mid p \text{ a place}\}$ that simulate each place and the transitions of a net.

Exercise 3.2 (Alternation Bounded QBF)

We define the following *alternation bounded* variants of QBF.

- $\Sigma_i\mathsf{QBF} = \{\psi \mid \psi = \exists\, \overline{x_1}\, \forall\, \overline{x_2} \ldots Q_i\, \overline{x_i}\, \varphi(\overline{x_1}, \ldots, \overline{x_i}) \text{ is true}\}$,
- $\Pi_i\mathsf{QBF} = \{\psi \mid \psi = \forall\, \overline{x_1}\, \exists\, \overline{x_2} \ldots Q_i\, \overline{x_i}\, \varphi(\overline{x_1}, \ldots, \overline{x_i}) \text{ is true}\}$,

where $\overline{x_j}$ denotes a finite sequence of variables and Q_i is a quantor. Note that there are at most $i-1$ alternations of quantors.

Prove by induction that $\Sigma_i\mathsf{QBF}$ ($\Pi_i\mathsf{QBF}$) can be decided by an alternating Turing Machine that runs in polynomial time, uses at most $i-1$ alternations between existential and universal states and branches first in an existential (universal) state.

Exercise 3.3 (Shortest Path)

Show that the following problem is in NL.

Shortest Path (SP)	
Input:	A directed graph $G = (V, E)$ and a $k \in \mathbb{N}$.
Question:	Does a shortest path from s to t have length exactly k?

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