Exercises to the lecture
Complexity Theory
Sheet 4

Prof. Dr. Roland Meyer
M.Sc. Peter Chini

Delivery until 26.11.2015 at 12h

Exercise 4.1 (Immerman and Szelepcsényi)

In the lecture we have shown that $\overline{PATH}$ is in NL. Use this to prove the theorem of Immerman and Szelepcsényi:
For $s : \mathbb{N} \to \mathbb{N}$ with $s(n) \geq log\, n$, we have:

$$\mathsf{NSPACE}(s(n)) = \text{co-}\mathsf{NSPACE}(s(n)).$$

Exercise 4.2 (Universal Turing Machine Part II)

Let t_2 be time constructible and $t_1^2 = o(t_2)$. Show that we have a strict inclusion:

$$\mathsf{DTIME}(t_1(n)) \subsetneq \mathsf{DTIME}(t_2(n)).$$

Hint: We have already shown that $\mathsf{DSPACE}(s_1) \subsetneq \mathsf{DSPACE}(s_2)$ *is a strict inclusion under suitable assumptions. Recall the proof of this theorem and use the same idea to prove the above result.*

Exercise 4.3 (Hierarchies and Padding)

Show the following statements, using the hierarchy and transfer results from the lecture:

a) $\mathsf{P} \subsetneq \mathsf{EXP}$,

b) $\mathsf{NL} \subsetneq \mathsf{PSPACE}$,

c) If $\mathsf{NL} = \mathsf{P}$ then we also have: $\mathsf{PSPACE} = \mathsf{EXP}$.

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