WS 2015/2016 25.11.2015

Exercises to the lecture
Complexity Theory
Sheet 5

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M.Sc. Peter Chini Delivery until 02.12.2015 at 12h

Exercise 5.1 (Reductions)

Let Σ_1 and Σ_2 denote two alphabets and let R be a set of functions from Σ_1^* to Σ_2^*.

a) Assume that A is a language in Σ_1^* and that $A \leq_m^R \Sigma_2^*$ holds. Show that $A = \Sigma_1^*$.

b) Let $|R| = 1$ and assume that we have languages $A, A' \subseteq \Sigma_1^*$ and $B \subseteq \Sigma_2^*$ so that:

$$A \leq_m^R B \text{ and } A' \leq_m^R B.$$

Show that we have $A = A'$.

Exercise 5.2 (Reductions and hardness)

Let C be a complexity class so that co-$C = C$ and let R be a set of functions. Assume that the language A is C-hard with respect to R-many-one reductions. Show that $\bar{A}$ is also C-hard with respect to R-many-one reductions.

Exercise 5.3 (Completeness in L)

Let Σ be a finite alphabet. Prove the following two statements:

a) A language A over Σ is in L if and only if $A \leq_m^{log} \{0,1\}$.

b) Any language A in L that satisfies $A \neq \emptyset$ and $A \neq \Sigma^*$ is already L-complete with respect to logspace-many-one reductions.

Exercise 5.4 (Acyclic reachability)

Show that we can reduce *PATH* to *ACYCLICPATH* with respect to logspace-many-one reductions. Conclude that $\overline{ACYCLICPATH}$ is NL-complete.

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