Concurrency Theory (SS 2015) Out: Mon, Apr 20 Due: Tue, Apr 28

Exercise Sheet 1

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Problem 1: Shared Memory Concurrency

(a) Let $\mathbf{load}$, $\mathbf{store}$ and $\mathbf{inc}$ be atomic operations for loading a variable's value into cache, storing a variable's cache value into memory and incrementing a variable's cache value by 1. If x is initially 0, argument why there are executions ending with $x = 2015$ of the program

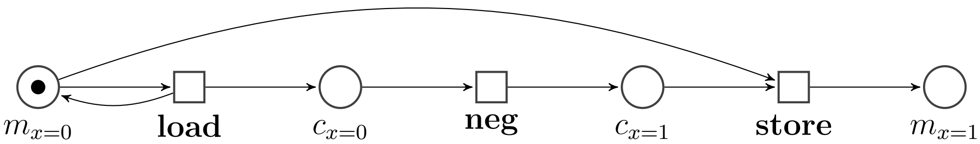
for $i = 1 \dots 2015$ **do**
$\mathbf{load}\ x$
$\mathbf{inc}\ x$
$\mathbf{store}\ x$
end for
	$\|$	**for** $i = 1 \dots 2015$ **do**
		$\mathbf{load}\ x$
		$\mathbf{inc}\ x$
		$\mathbf{store}\ x$
		end for

Misleading Hint: Each of the two above programs is basically incrementing x 2015 times.

(b) Let $\mathbf{neg}$ be another atomic operation for negating a **Boolean** variable's cache value. If x is initially 0, give a Petri net representation of the following program and specify transition sequences which describe why the program may terminate with $x = 0$ or $x = 1$ in memory.

$\mathbf{load}\ x$ $\mathbf{neg}\ x$ $\mathbf{store}\ x$	$\|$	$\mathbf{load}\ x$ $\mathbf{neg}\ x$ $\mathbf{store}\ x$

Hint: Without concurrency, the net for one half of the above program could be represented by



Problem 2: Boundedness and Termination

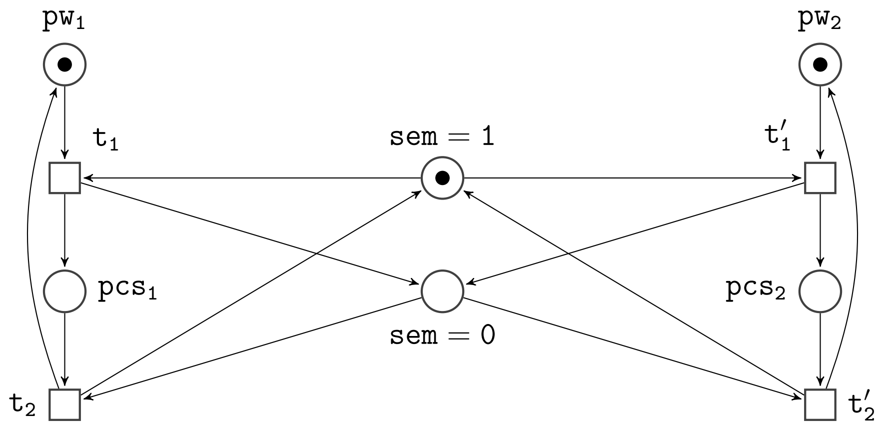
Give Petri nets $\mathcal{N}_{b\wedge t}$, $\mathcal{N}_{b\wedge\neg t}$, $\mathcal{N}_{\neg b\wedge t}$ and $\mathcal{N}_{\neg b\wedge\neg t}$ such that

- $\mathcal{N}_{b\wedge t}$ is bounded and terminating
- $\mathcal{N}_{b\wedge\neg t}$ is bounded and not terminating
- $\mathcal{N}_{\neg b\wedge t}$ is unbounded and terminating
- $\mathcal{N}_{\neg b\wedge\neg t}$ is unbounded and not terminating.

If one of the Petri nets above does not exist, prove why that is the case.

Problem 3: Invariants for Mutual Exclusion

Recall the mutual exclusion protocol given in class:



As seen in class $(0\ 1\ 0\ 1\ 2\ 1)^T$ is an invariant used to prove no marking M with

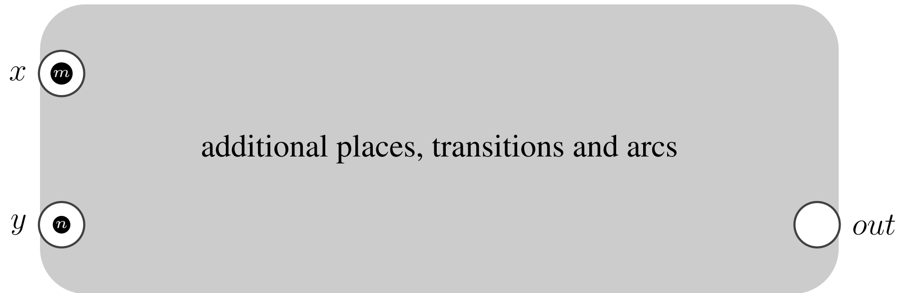
$$M(\texttt{pcs}_1) = M(\texttt{pcs}_2) = M(\texttt{sem} = 0) = 1$$

is reachable.

Which invariants can be used to prove no marking M with just $M(\texttt{pcs}_1) = M(\texttt{pcs}_2) = 1$ is reachable? Give a general description and prove mutual exclusion for such a concrete invariant.

Problem 4: "Multiplication" as a Petri Net

Consider the Petri net which contains places x, y and out as in the picture below.



Add places (and corresponding tokens), transitions and arcs to the net such that for arbitrary $m, n \in \mathbb{N}$, if $M_0(x) = m$, $M_0(y) = n$ and $M_0(out) = 0$ the net always reaches a deadlock and, if M_{ter} is the deadlock marking, $M_{ter}(out)$ is **any** of $\{0, \ldots, m \cdot n\}$.

Argument the correctness of your construction.