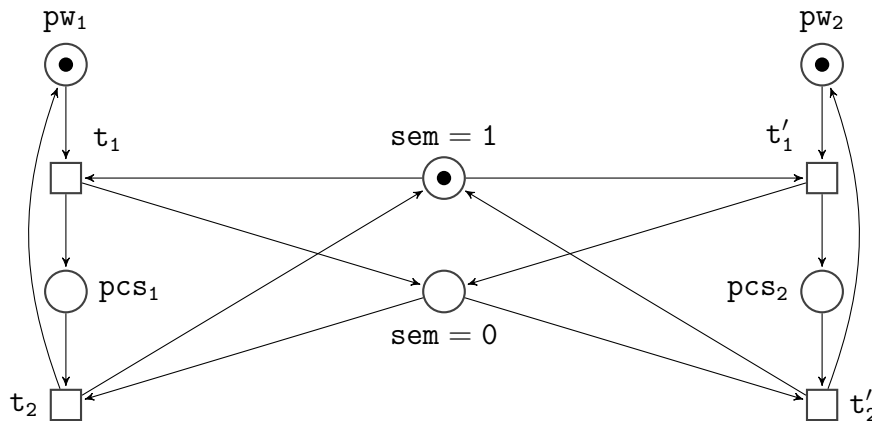


Exercise Sheet 4

Problem 1: Invariants for Mutual Exclusion

Consider the following mutual exclusion protocol:



Notice that $(0 \ 1 \ 0 \ 1 \ 2 \ 1)^T$ is an invariant used to prove no marking M with

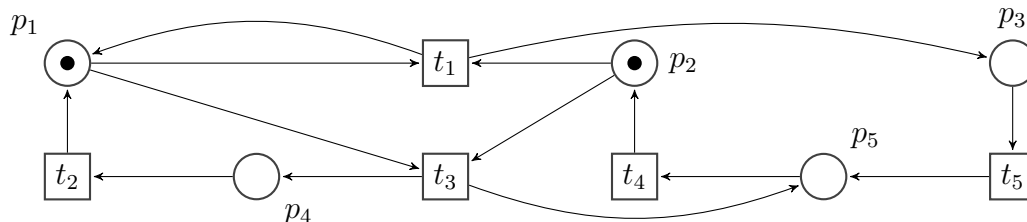
$$M(pcs_1) = M(pcs_2) = M(sem = 0) = 1$$

is reachable.

Which invariants can be used to prove no marking M with just $M(pcs_1) = M(pcs_2) = 1$ is reachable? Give a general description and prove mutual exclusion for such a concrete invariant.

Problem 2: Simple Mutual Exclusion

Consider the Petri net N given below:



- (a) Compute the set of structural invariants.
- (b) Determine the set of all traps.
- (c) Using (a) and (b), prove that p_3 and p_4 are mutually exclusive.

Problem 3: S/T-Invariants for Petri Nets

Let $N = (S, T, W)$ be a Petri net.

- (a) Prove that if I and J are structural (S-) invariants of N , so are $I + J$ and $k \cdot I$ ($\forall k \in \mathbb{Z}$).
- (b) Prove that if I and J are transition (T-) invariants of N , so are $I + J$ and $k \cdot I$ ($\forall k \in \mathbb{N}$).
- (c) Give a Petri net that is acyclic but has a transition (T-) invariant.