Exercise Sheet 8

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Problem 1: Cops and Robbers

Given a tree T, how many cops are necessary to catch the robber?

a) Give a winning strategy for the cops and argue its correctness.

b) Give a strategy for the robber that is winning if there are less cops than necessary.

Problem 2: MSO Formulas

a) Give an MSO formula that holds if and only if the domain is infinite. Argue correctness.

b) Given a finite graph G, give an MSO formula φ such that φ holds on the structure induced by G if and only if G contains a clique of size k.

c) Given a finite graph G, give a MSO formula φ such that φ holds on the structure induced by G if and only if G is a tree (assume the edges of G are labelled with labels in $\{c_1, \ldots, c_n\}$ with the intended meaning of c_i being "i-th child").

Problem 3: MSO-Interpretations

Consider the class $\mathcal{T}_n$ consisting of all the infinite complete trees where every node has n children. From any node x, its i-th child is the (only) node y such that there is a (directed) edge (x, y) labelled with c_i.

a) Propose an encoding of elements of $\mathcal{T}_n$ as structures of MSO.

b) Show an MSO-interpretation of structures representing trees in $\mathcal{T}_3$ into structures representing trees in $\mathcal{T}_2$.

c) Generalise the above interpretation to an MSO-interpretation of $\mathcal{T}_2$ in $\mathcal{T}_n$ for any fixed n.

Problem 4: Grids and Trees

Consider the $n \times 3$ grid $\mathcal{G}_{n \times 3}$ as below
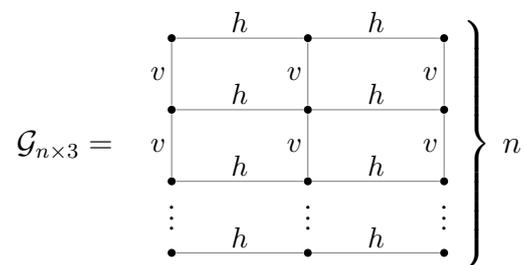
Here the labels h and v are used for horizontal and vertical edges respectively.

Consider the complete 4-ary tree of height n, $\mathcal{H}_{4,n}$, where the labels a, b, c and d are used to label the edges to the first to fourth child respectively.

a) Give an MSO-interpretation of $\mathcal{G}_{n \times 3}$ into $\mathcal{H}_{4,n}$.

b) Does the interpretation generalise to the grid $\mathcal{G}_{\omega \times 3}$ with 3 columns and infinitely many rows?

c) Rabin's Theorem tells us that the MSO-theory of the binary tree is decidable. Given that we know that the MSO-theory of the grid with infinitely many rows and columns $\mathcal{G}_{\omega \times \omega}$ is undecidable, what can we infer about the existence of MSO-interpretations of $\mathcal{G}_{\omega \times \omega}$ into n-ary infinite trees?