Concurrency Theory (SS 2015) Out: Wed, June 17 Due: Tue, June 23

Exercise Sheet 9

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Problem 1: Colorings for Tree decompositions

a) Recall that a *graph coloring* is a map from nodes of the graph to a set of colors C such that every couple of adjacent nodes are assigned two different colors. Prove the following lemma from the lecture by giving a coloring algorithm and proving its correctness: Given a graph with a tree decomposition of width m, there is a graph coloring with $2m + 2$ colors.

b) Recall the MSO-interpretation of a formula $x = y$ on graphs to a formula $\psi_=(x, y)$ on the extended tree decomposition of the graph described in the lectures. Prove that $\psi_=(x, y)$ holds on two nodes of the extended tree decomposition of G if and only if $x = y$ holds in G on the corresponding nodes.

Problem 2: MSO-Definability of Tree Decompositions

Given a tree decomposition T of a graph G, we call T' the *extended tree decomposition* of G, where T' is defined as in the lecture. We represent an extended tree decomposition with structures $S_{T'}$ with signature

$$(A, P'_N, P'_\rightarrow, P'_V, P'_E, (P'_a)_{a\in\Sigma_V}, (Q'_a)_{a\in\Sigma_E}, (P'_c)_{c\in C}, (R'_i)_{1\leq i\leq\binom{m+1}{2}})$$

Give a MSO formula φ such that $S_{T'} \models \varphi$ if and only if T' is an extended tree decomposition (of any graph). *Hint: you only need to check that the relations are consistent.*

Problem 3: From Automata to MSO

Show a translation from non-deterministic finite automata to MSO formulas on words so that the set of models of the translation of an automaton coincides with the language of the automaton.

Problem 4: Büchi's Construction on Trees

Consider the translation from MSO formulas on words to finite automata discussed in the lecture. We want to show that the technique naturally extends to finite ranked trees. For simplicity, we restrict to the case where every letter in the alphabet has the same arity k, i.e. every node has exactly k children. For this exercise, we represent finite trees with nodes labelled by k-ary letters from a finite alphabet Σ, with structures in the signature $(N, (P_a)_{a\in\Sigma}, (C_i)_{1\leq i\leq k})$; N is the set of nodes, $P_a(x)$ means node x is labelled with a and $C_i(x, y)$ means that x is the i-th child of y.

a) Describe informally how to modify Büchi's construction in order to generalise it to formulas on trees.

b) Define an MSO formula ρ_a that is true on a tree if and only if the root of the tree is labelled with a.

c) Use the procedure you described at point a) to derive a tree automaton A from ρ_a such that ρ_a holds on a tree T if and only if the automaton A accepts T.