Concurrency Theory (SS 2015) Out: Wed, 1 Jul Due: Tue, 7 Jul

Exercise Sheet 11

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Problem 1: Hardest Language

A language L_0 is a *hardest language* for a language class $\mathcal{C}$ if for every $L \in \mathcal{C}$ there exists an homomorphism α such that $L = \alpha^{-1}(L_0)$.

For any regular language L, let $\|L\|$ be the minimum number of states of an automaton generating L, i.e. $\|L\| = \min\{|Q| \mid A = (Q, E, q_0, F), L = \mathcal{L}(A)\}$. Show that:

a) $L = \alpha^{-1}(L')$ implies $\|L\| \leq \|L'\|$.

b) For each $n \in \mathbb{N}$, there exists a regular language with $\|L\| > n$.

c) Use 1.a and 1.b to show that there is no hardest language for the class of regular languages.

Problem 2: Concatenation

a) Show that if $L' \subseteq X^*$ is closed under concatenation and $L = \alpha^{-1}(L')$, then L is closed under concatenation.

b) Show that D'_2 is not a hardest language for CFL.

Problem 3: Kleene Iteration

Let $L \subseteq X^*$. Show that the full trio generated by $(L\#)^*$ is closed under Kleene iteration.

Problem 4: Generating languages

Let $M = \{a^n b^n \mid n \in \mathbb{N}\}$ and $\mathcal{C}_M$ be the full trio generated by M.

a) Let $R_n = \{w\overline{w} \mid w \in X_n^*\}$ where $\overline{w}$ is the word w with each occurrence of each letter x_i replaced by the letter $\overline{x_i} \in \overline{X}_n$.
Show that for each $n \in \mathbb{N}$, the complement of R_n is in $\mathcal{C}_M$.

b) Sketch how you can adapt the solution of the previous problem to prove that for each homomorphism $\alpha\colon X^* \to Y^*$, with $X \cap Y = \emptyset$, the complement of the language $\{w\alpha(w) \mid w \in X^*\}$ is in $\mathcal{C}_M$.