Concurrency Theory (SS 2015) Out: Wed, 15 Jul Due: Tue, 21 Jul

Exercise Sheet 13

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Problem 1: Q_2 and Palindromes

Recall the definition of $Q_\alpha := \{w\alpha(w^R) \mid w \in X^*\}$ for an homomorphism $\alpha\colon X^* \to Y^*$, and $Q_n := \{w\gamma_n(w^R) \mid w \in {X_n}^*\}$ where $\gamma_n\colon {X_n}^* \to \overline{X_n}^*$ is the homomorphism with $\gamma_n(a_i) = \overline{a_i}$.

a) Let $\mathcal{C}$ be a full trio. Show that the following statements are equivalent:

- $\mathcal{C}$ contains Q_2.
- $\mathcal{C}$ contains Q_α for every homomorphism $\alpha : X^* \to Y^*$.

[Hint: *Show that if $\mathcal{C}$ contains Q_2, then it contains Q_n for every $n \in \mathbb{N}$.*]

b) Show that $\mathcal{C}$ contains Q_2 if and only if it contains the set $\left\{ w \in \{a,b\}^* \;\middle|\; w = w^R \right\}$ of palindromes.

Problem 2: Permutations

a) Show that $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ is a Petri net language.

b) For a language L we define $\Pi(L) := \{w \mid \exists w' \in L : w \text{ is a permutation of } w'\}$.
Show that if L is a Petri net language, $\Pi(L)$ is a Petri net language as well.

c) Show that if L is a context-free language, $\Pi(L)$ is not necessarily context-free.

Problem 3: Chomsky-Schützenberger and D_1'

a) Show that the Petri net languages form the smallest full trio that contains D_1' and is closed under intersection.

b) Deduce from 3a) that not every context-free language can be written as $\beta(\alpha^{-1}(D_1') \cap K)$ with homomorphisms α, β and regular language K.

Problem 4: Lossy Channel Languages

Recall the definition of a Lossy Channel system from the lecture.
For an LCS $S' = (Q, q_0, C, M, \Delta)$ (where $\Delta \subseteq Q \times OP \times Q$ is the finite set of transitions) we define, as we did for Petri Nets, its X-labelled version $S = (Q, q_0, C, M, \Delta, X, \lambda, F)$ by means of a labelling function $\lambda \colon \Delta \to X \cup \{\varepsilon\}$ which associates to every transition a label or the empty word, and final states $F \subseteq Q$. We write $(q_1, W_1) \to_t (q_2, W_2)$ when the transition $t \in \Delta$ generates a transition between two configurations $(q_1, W_1') \to (q_2, W_2')$ with $W_1' \preceq^* W_1$ and $W_2 \preceq^* W_2'$. We denote by ε^C the function associating to each channel in C the empty word. Then the language generated by S is defined as

$$\mathcal{L}(S) := \left\{ \lambda(t_1) \cdots \lambda(t_n) \;\middle|\; (q_0, \varepsilon^C) \to_{t_1} \cdots \to_{t_n} (q_n, \varepsilon^C), q_n \in F \right\}.$$

The LCS languages are precisely those that can be generated by an LCS. Given these definitions, answer the following:

a) Show that LCS languages form a full trio.

b) Show that LCS languages are also closed under intersection and Kleene star.

c) Use the following theorem to deduce that $\{a^n b^n \mid n \in \mathbb{N}\}$ is not an LCS language:

> **Theorem** *(Hartmanis & Hopcroft). The class of the recursively enumerable languages is the smallest full trio containing $\{a^n b^n \mid n \in \mathbb{N}\}$ that is closed under intersection and Kleene star.*