SS 2024 02.05.2024

Übungen zur Vorlesung
Concurrency Theory
Blatt 1

Prof. Dr. Roland Meyer
Jan Grünke Abgabe bis 14.05.2024 um 23:59 Uhr

**Aufgabe 1.1** (Kirchhoff Equations)

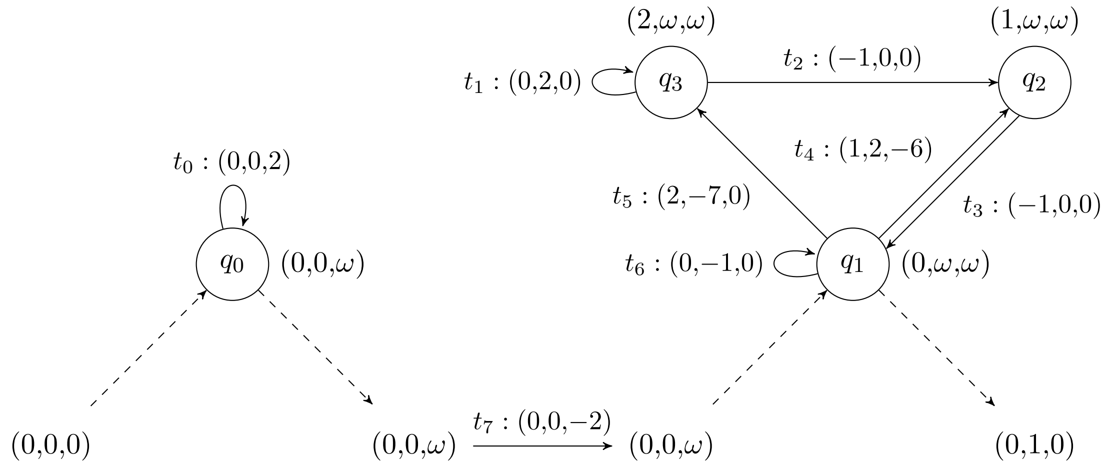
Let $G = (V, E)$ be a directed graph and $c$ a cycle in $G$. Show that $\psi(c)$ is a solution of the Kirchhoff equation $\Sigma_{e=(-,v)} x(e) - \Sigma_{e=(v,-)} x(e) = 0$ for all vertices $v \in V$.

**Aufgabe 1.2** (Reducing Petri net reachability to MGTS intermediate acceptance)

Let $N = (S, T, W)$ be a Petri net and $M_0, M_f \in \mathbb{N}^{|S|}$ be markings. Give a MGTS that has an intermediate accepting $\mathbb{N}$-run if and only if $M_f$ is reachable from $M_0$ in $N$.

**Aufgabe 1.3** (VASS reachability)

Consider the following MGTS $W = G_0.t_7.G_1$:




**(a)** Write down the characteristic equation $Char(W)$. You may replace constant variables with their value and omit equations that are always true.

**(b)** Show that $W$ is perfect. In particular, give up- and down-pumping sequences for $G_0$ and $G_1$ and show that the support justifies the unboundedness.

**(c)** Give a full support solution $s_h$ of the homogeneous variant of $Char(W)$.

**(d)** Give a $\mathbb{Z}$-run $\rho$. For this, find a solution $s_c$ of $Char(W)$ and add your full support solution $s_h$ if necesssary.

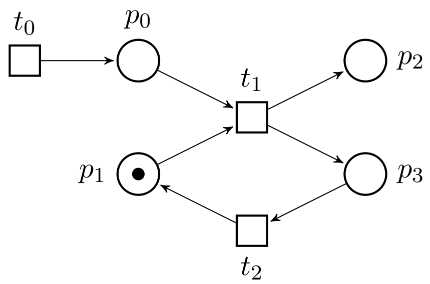
**(e)** The $\mathbb{Z}$-run from (d) has the form $\rho = \rho_0.t_7.\rho_1$. Use Lambert's iterations lemma to get an $\mathbb{N}$-run for $\rho_1$. For this embed the up-pumping sequence $u_1$ and the down-punping sequence $v_1$ for $G_1$ from (b) in the support solution $s_h$ from (c). In particular, find $m \in \mathbb{N}$ such that:

$$\begin{aligned} m \cdot s_h[T(G_1)] - \psi(u_1) - \psi(v_1) &\geq 1 \\ m \cdot s_h[G_1, in, 3] + \mathit{eff}(u_1)(3) &\geq 1 \\ m \cdot s_h[G_1, out, 3] - \mathit{eff}(v_1)(3) &\geq 1 \end{aligned}$$

Then, give a $\mathbb{Z}$-run $u_1.w_1.v_1$ with Parik image $m \cdot s_h[T(G_1)]$. Finally, this run can be pumped such that $u_1^k.\rho_1.w_1^k.v_1^k$ is a $\mathbb{N}$-run. Give a sufficient $k \in \mathbb{N}$ and the resulting run.

**Aufgabe 1.4** (Abdulla's backwards search for Petri nets)

Consider the following Petri net:




**(a)** Write the definition of $\mathit{minpre}(M)$ for Petri nets. Is it computable?

**(b)** Run the backwards search to prove that the marking $M = (0\ 0\ 2\ 0)^T$ is coverable.

**Abgabe bis 14.05.2024 um 23:59 Uhr per Mail an jan.gruenke1@tu-bs.de.**