

Übungen zur Vorlesung  
Concurrency Theory  
Blatt 2

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Abgabe bis 14.05.2024 um 23:59 Uhr

**Aufgabe 2.1** (Kirchhoff Equations)

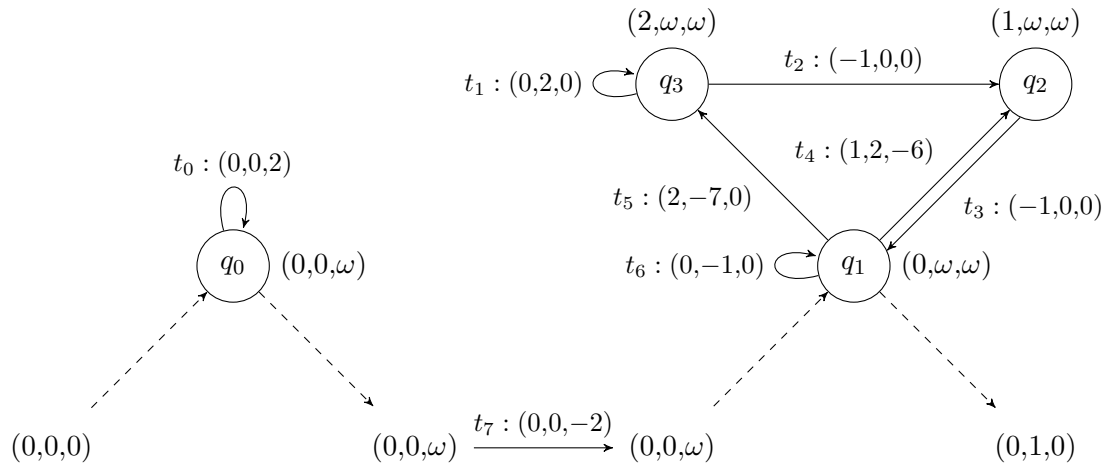
Let  $G = (V, E)$  be a directed graph and  $c$  a cycle in  $G$ . Show that  $\psi(c)$  is a solution of the Kirchhoff equation  $\sum_{e=(-,v)} x(e) - \sum_{e=(v,-)} x(e) = 0$  for all vertices  $v \in V$ .

**Aufgabe 2.2** (Reducing Petri net reachability to MGTS intermediate acceptance)

Let  $N = (S, T, W)$  be a Petri net and  $M_0, M_f \in \mathbb{N}^{|S|}$  be markings. Give a MGTS that has an intermediate accepting  $\mathbb{N}$ -run if and only if  $M_f$  is reachable from  $M_0$  in  $N$ .

**Aufgabe 2.3** (VASS reachability)

Consider the following MGTS  $W = G_0.t_7.G_1$ :



- (a) Write down the characteristic equation  $Char(W)$ . You may replace constant variables with their value and omit equations that are always true.
- (b) Show that  $W$  is perfect. In particular, give up- and down-pumping sequences for  $G_0$  and  $G_1$  and show that the support justifies the unboundedness.
- (c) Give a full support solution  $s_h$  of the homogeneous variant of  $Char(W)$ .
- (d) Give a  $\mathbb{Z}$ -run  $\rho$ . For this, find a solution  $s_c$  of  $Char(W)$  and add your full support solution  $s_h$ .

- (e) The  $\mathbb{Z}$ -run from (d) has the form  $\rho = \rho_0.t7.\rho_1$ . Use Lambert's iterations lemma to get an  $\mathbb{N}$ -run for  $\rho_1$ . For this embed the up-pumping sequence  $u_1$  and the down-pumping sequence  $v_1$  for  $G_1$  from (b) in the support solution  $s_h$  from (c). In particular, find  $m \in \mathbb{N}$  such that:

$$m \cdot s_h[T(G_1)] - \psi(u_1) - \psi(v_1) \geq 1$$

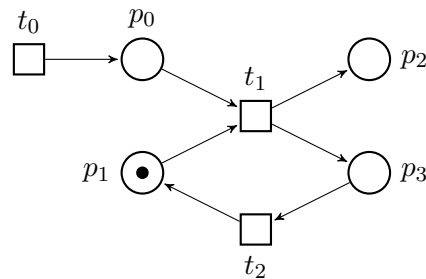
$$m \cdot s_h[G_1, in, 3] + eff(u_1)(3) \geq 1$$

$$m \cdot s_h[G_1, out, 3] - eff(v_1)(3) \geq 1$$

Then, give a  $\mathbb{Z}$ -run  $u_1.w_1.v_1$  with Parik image  $m \cdot s_h[T(G_1)]$ . Finally, this run can be pumped such that  $u_1^k.\rho_1.w_1^k.v_1^k$  is a  $\mathbb{N}$ -run. Give a sufficient  $k \in \mathbb{N}$  and the resulting run.

### Aufgabe 2.4 (Abdulla’s backwards search for Petri nets)

Consider the following Petri net:



- (a) Write the definition of  $minpre(M)$  for Petri nets. Is it computable?
- (b) Run the backwards search to prove that the marking  $M = (0 \ 0 \ 2 \ 0)^T$  is coverable.

Abgabe bis 14.05.2024 um 23:59 Uhr per Mail an [jan.gruenke1@tu-bs.de](mailto:jan.gruenke1@tu-bs.de).