Concurrency Theory (WS 2010/11) Out: Wed, Dec 1 Due: Mon, Dec 6

Exercise Sheet 6

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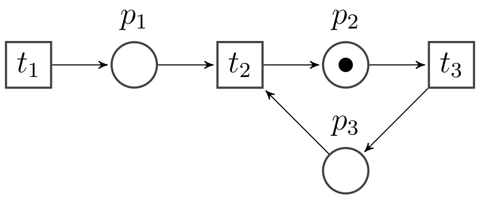
Problem 1: Decision Procedure for Place Boundedness

Consider a Petri net $N = (S, T, W, M_0)$ and let $Cov(N) = (V, E, M_0)$ be its coverability graph. Prove that $s \in S$ is unbounded if and only if there exists $M_\omega \in V$ with $M_\omega(s) = \omega$.

Problem 2: Karp & Miller Algorithm

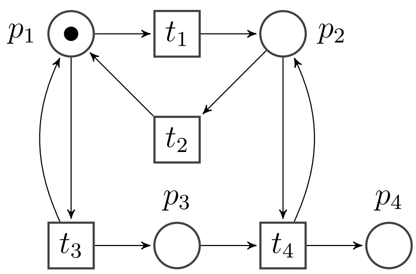
Construct the coverability graph for the following Petri net:



Specify the order in which $V,\ L,\ E$ are changed and why (or why not) new nodes are created.

Problem 3: Coverability and Place Boundedness

Construct the coverability graph for the following Petri net:



(a) Name the unbounded places in the net by specifying all the nodes in the coverability graph which allow you to deduce their unboundedness.

(b) For each of the following markings of the Petri net:

$$(1\,0\,0\,0)^T,\ (0\,0\,1\,0)^T,\ (1\,1\,0\,0)^T,\ (0\,0\,1\,1)^T,\ (1\,0\,1\,0)^T,\ (0\,1\,0\,1)^T$$

specify all the nodes in the coverability graph (if any) that cover them.

Problem 4: Proof of Lemma - t *introduces new* ω*'s*

Let $N = (S, T, W, M_0)$ be a Petri net with coverability graph $Cov(N) = (V, E, M_0)$ and let $M_0 \xrightarrow{\sigma} M_\omega^n \xrightarrow{t} M_\omega^{n+1}$ for some $\sigma \in T^n$ and $t \in T$.

Assume that for all $k \in \mathbb{N}$ there exists $M \in R(N)$ such that

$$\begin{cases} M(s) \geq k & \text{if } s \in \Omega(M_\omega^n) \\ M(s) = M_\omega^n(s) & \text{if } s \in S \setminus \Omega(M_\omega^n). \end{cases}$$

Fix $k_0 \in \mathbb{N}$ and assume $|\Omega(M_\omega^{n+1})| = |\Omega(M_\omega^n)| + 1$. Prove that there is $M' \in R(N)$ such that

$$\begin{cases} M'(s) \geq k_0 & \text{if } s \in \Omega(M_\omega^{n+1}) \\ M'(s) = M_\omega^{n+1}(s) & \text{if } s \in S \setminus \Omega(M_\omega^{n+1}). \end{cases}$$