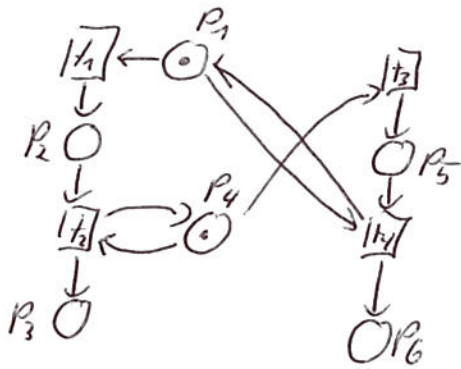


The following Petri net models a simplified variant of Dekker's mutual exclusion protocol.



- Set up the marking equation.
- Places P_3 and P_6 represent critical sections of the left and the right program, respectively. Set up a co-linear property that reflects mutual exclusion.
- Construct the trap matrix of the Petri net.
- Use your findings in (a) to (c) to construct the corresponding enhanced verification system. Prove its infeasibility.

(a)

$$M = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} t_1 & t_2 & t_3 & t_4 \\ -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} X$$

with M, X integer, $M, X \geq 0$

(b) $(0 \ 0 \ 1 \ 0 \ 0 \ 1) M \geq 2$

(c)

$$C_Q = \begin{pmatrix} p_1 t_1 & p_2 t_2 & p_4 t_2 & p_4 t_3 & p_5 t_4 & p_1 t_4 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(d) Enhanced verification system:

$$M = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} X \quad \begin{matrix} (L_1) \\ (L_2) \\ \vdots \\ (L_6) \end{matrix}$$

$$M, X \geq 0$$

$$(0 \ 0 \ 1 \ 0 \ 0 \ 1) M \geq 2$$

$$M \geq z \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} z$$

$$z \geq 0$$

$$z \geq 0$$

Argue for infeasibility:

By (L_1) , we have $x_{t_1} \leq 1$

By this + (L_2) , we get $x_{t_2} \leq 1$.

Similarly, $x_{t_3} \leq 1$ and $x_{t_4} \leq 1$.

To reach $(001001) \mu \geq 2$,
we need

$$x_{t_2} = 1 \text{ and } x_{t_4} = 1.$$

This entails $(L_2) + (L_5)$

$$x_{t_1} = 1 \text{ and } x_{t_3} = 1.$$

Hence,

$$\mu = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Put into trap equation:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \geq 2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} z \quad \begin{array}{l} (L_1') \\ \vdots \\ (L_6') \end{array}$$

By (L_2') ,

$$z_1 \leq z_2.$$

Similarly, by (L_5')

$$z_4 \leq z_5$$

By (L_1) ,

$$z_1 > z_5$$

By (L_4') ,

$$z_4 > z_2.$$

We conclude:

$$z_1 > z_5 \geq z_4 > z_2.$$

A contradiction to the requirement that $z_1 \leq z_2$

The system is infeasible, mutual exclusion holds. □