Concurrency Theory (WS 2011/12) Out: Tue, Nov 1 Due: Mon, Nov 7

Exercise Sheet 3

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Problem 1: Marking Equation

Let $N = (S, T, \mathbb{F}, \mathbb{B})$ be a Petri net with connectivity matrix $\mathbb{C}$ and $M_1, M_2 \in \mathbb{N}^{|S|}$, $\sigma \in T^*$ such that $M_1[\sigma\rangle M_2$. Prove that $M_2 = M_1 + \mathbb{C} \cdot p(\sigma)$, where $p(\bullet)$ is the *Parikh image* function.

Hint: $\mathbb{C}(\bullet, t) = \mathbb{C} \cdot E_t$*, where* E_t *is the unit vector having* 1 *at position* t *and* 0 *elsewhere.*

Problem 2: Place Boundedness via S-Invariants

Let s be a place of the marked net $N = (S, T, W, M_0)$. Prove that if there is a nonegative S-invariant I satisfying $I(s) \geq 1$ then s is bounded by

$$\frac{1}{I(s)} \sum_{s' \in S} I(s') \cdot M_0(s').$$

Problem 3: Lemma on Structural Boundedness

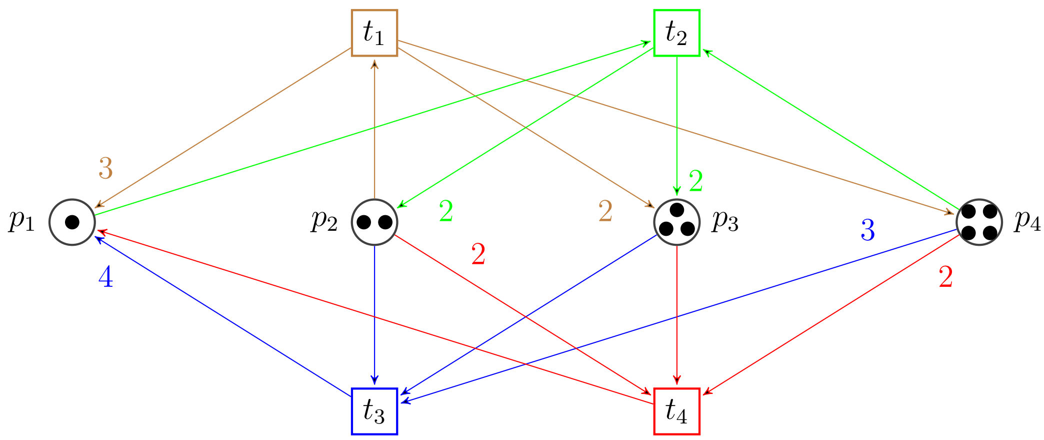
Prove that if the Petri net $N = (S, T, W, M_0)$ is unbounded then $\mathbb{C}x \gneq 0$ has a solution in $\mathbb{N}^{|T|}$.

Problem 4: More on S/T-Invariants

Let $N = (S, T, W)$ be a Petri net.

(a) Prove N's transition (T-) invariants form a vector space, i.e., prove that if I and J are T-invariants so are $I + J$ and $k \cdot I$ for $k \in \mathbb{Z}$.

(b) Compute a basis of S-invariants for the net:



Note: the different colors are used only for not mixing the different numerical labels of the arcs.