Concurrency Theory (WS 2011/12)

Out: Tue, Nov 8 Due: Mon, Nov 14

Exercise Sheet 4

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Problem 1: Simple Mutual Exclusion

Consider the Petri net N given below:



- (a) Compute the set of structural invariants.
- (**b**) Determine the set of all traps.
- (c) Using both (a) and (b), prove that p_3 and p_4 are mutually exclusive.

Problem 2: Deadlock-Free Petri Net

Consider the Petri net $N = (S, T, W, M_0)$ depicted below and let \mathbb{C} be its connectivity matrix.



(a) Describe the set $D \subseteq \mathbb{N}^{|S|}$ of markings which represent deadlocks of N and determine whether $\mathbb{C}\mathbf{x} = M - M_0$ is solvable in $\mathbb{N}^{|T|}$ for some $M \in D$.

(b) Determine the S-invariants and the traps of the Petri net N.

(c) Prove that $D \cap R(N) = \emptyset$ by showing that the system of linear inequalities describing D together with N's traps and S-invariants is inconsistent. Are **all/any** traps really needed?

Problem 3: Towards the Enhanced Verification System

Consider the following Petri net:



- (a) Give the marking equation and a co-linear property reflecting deadlock-freedom.
- (b) Set up the trap matrix and enumerate the traps of the Petri net.

Problem 4: Family of Generating Traps

Add arcs to the Petri net N below so that its family of generating traps contains exponentially (in N's size) many traps. Once added, describe N = (S, T, W) formally and prove that the family of generating traps is exponential in N's size.



Does the generating family of traps for N contain only minimal traps? Argument your answer.