Concurrency Theory (WS 2016) Out: Thu, 27 Oct Due: Tue, 02 Nov

Exercise Sheet 1

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Problem 1: Some Proofs

Let $N = (S, T, W, M_0)$ be a Petri net and $M, M_1, M_2 \in \mathbb{N}^S$ be markings of N.

a) Prove that $\mathcal{R}(N)$ is finite if and only if every place of N is k-bounded, for some $k \in \mathbb{N}$.

b) Prove that $\forall \sigma \in T^*$: if $M_1 \xrightarrow{\sigma} M_2$ then $(M_1 + M) \xrightarrow{\sigma} (M_2 + M)$.

Problem 2: Boundedness and Termination

Give Petri nets $N_{b\wedge t}$, $N_{b\wedge\neg t}$, $N_{\neg b\wedge t}$ and $N_{\neg b\wedge\neg t}$ such that

- $N_{b\wedge t}$ is bounded and terminating.
- $N_{b\wedge\neg t}$ is bounded and not terminating.
- $N_{\neg b\wedge t}$ is unbounded and terminating.
- $N_{\neg b\wedge\neg t}$ is unbounded and not terminating.

If one of the Petri nets above does not exist, argue why that is the case.

Problem 3: Vector Addition Systems

A *vector addition system* (VAS) of *dimension* k is a tuple (v, A) where $v \in \mathbb{N}^k$ is the *start vector* and $A \subset \mathbb{Z}^k$ is a finite *addition set*. A *path* of length n in (v, A) is an element $\sigma_0, \ldots, \sigma_n \in (\mathbb{N}^k)^*$ where $\sigma_0 = v$ and $\sigma_i - \sigma_{i-1} \in A$ for all $i = 1, \ldots, n$. A vector $w \in \mathbb{N}^k$ is reachable in (v, A) if there exists a path ending in w.

A *vector addition system with states* (VASS) of *dimension* k is a tuple (Q, q_0, v, A) where Q is a finite set of *states*, $q_0 \in Q$ is the *initial state*, $v \in \mathbb{N}^k$ is the *start vector* and $A \subset Q \times \mathbb{Z}^k \times Q$ is a finite set of *transitions*. A *path* of length n in (Q, q_0, v, A) is an element $(s_0, v_0), \ldots, (s_n, v_n) \in (Q \times \mathbb{N}^k)^*$ where $(s_n, v_0) = (q_0, v)$ and $(s_{i-1}, v_i - v_{i-1}, s_i) \in A$ for all $i = 1, \ldots, n$. A configuration $w \in Q \times \mathbb{N}^k$ is reachable if there exists a path ending in w.

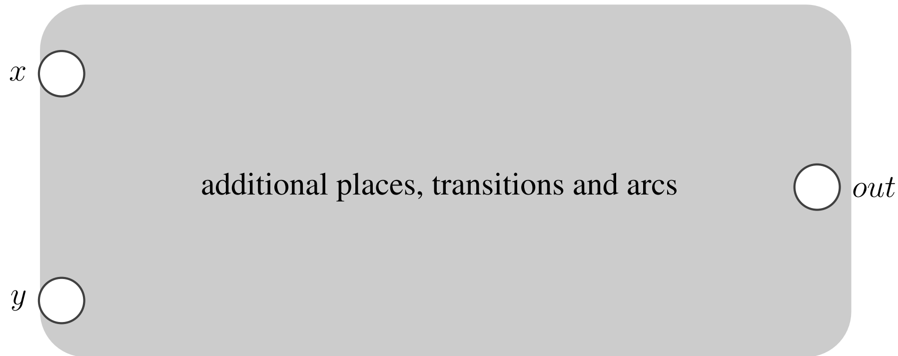
a) Reduce reachability in Petri nets to reachability in VASS.

b) Reduce reachability in VASS to reachability in VAS.

c) Reduce reachability in VAS to reachability in Petri nets.

Problem 4: Addition and Multiplication as Petri Nets

Consider the Petri net template which contains places x, y and out as in the picture below:



Add places and transitions to your liking to obtain two Petri nets N_a and N_m such that:

a) if $M_0(x) = m$, $M_0(y) = n$ and $M_0(out) = 0$, N_a always terminates with marking M_{ter} such that $M_{ter}(out) = m + n$.

b) if $M_0(x) = m$, $M_0(y) = n$ and $M_0(out) = 0$, N_m always terminates with marking M_{ter} such that $M_{ter}(out)$ is any of $\{0, \ldots, m \cdot n\}$.

In both cases provide the initial marking of the additional places you might have added and argument why your Petri nets do what they should.

c) We want to enforce that Petri net of (b) always ends with $M_{ter}(out) = m \cdot n$. Describe an extension of Petri nets with which you can achieve this.

Problem 5: Invariants (Optional)

Propose a condition on the transitions of a Petri net that guarantees that the total token count will be constant in each reachable marking. Is your condition also necessary? Can you make it necessary if not?