Concurrency Theory (WS 2016) Out: Thu, 10 Nov Due: Wed, 16 Nov

Exercise Sheet 3

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Problem 1: Structural Boundedness

Let $N = (S, T, \mathbb{F}, \mathbb{B})$ be a Petri net with connectivity matrix $\mathbb{C}$. N is *structurally bounded* if it is bounded from every initial marking.

a) Let $M \in \mathbb{N}^{|S|}$. Prove that there is a marking $M_1 \in \mathbb{N}^{|S|}$ with $M_1 + M \in \mathcal{R}(M_1)$ if and only if $M = \mathbb{C} \cdot x$ has a solution in $\mathbb{N}^{|T|}$.

b) Prove that there is an initial marking M_0 so that (N, M_0) is unbounded if and only if $\mathbb{C} \cdot x \gneqq 0$ has a solution in $\mathbb{N}^{|T|}$.

c) Let $I \in \mathbb{N}^{|S|}$ be a structural invariant and $s \in S$ with $I(s) > 0$. Show that s is bounded under any initial marking $M_0 \in \mathbb{N}^{|S|}$.

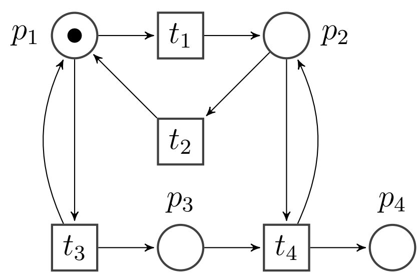
Problem 2: Reductions

a) Reduce the coverability problem to the reachability problem.

b) Adapt the algorithm for boundedness to decide termination.

Problem 3: Coverability and Place Boundedness

Consider the following marked Petri net N:



a) Construct the coverability graph $\mathrm{Cov}(N)$ using the algorithm seen in the lecture.

Recall that with $\mathrm{Cov}(N)$ we can solve any coverability problem instance.

b) Is $\mathrm{Cov}(N)$ unique?

c) Do you need the edges of $\mathrm{Cov}(N)$ to solve a coverability instance?

d) Do you need all the markings in the graph to solve any coverability instance?

Problem 4: Termination and Correctness

Consider a Petri net $N = (S, T, W, M_0)$ and prove the following claims:

a) The Karp-Miller algorithm of the lecture (which computes the coverability graph) terminates.

b) If $M_0 \xrightarrow{\sigma} M$ with $\sigma \in T^*$, then there exists some $L \in \mathbb{N}_\omega^{|S|}$ such that $M_0 \rightsquigarrow^* L$ in $\mathrm{Cov}(N)$ and $L \geq M$.

c) **OPTIONAL:** Assume $M_1 \xrightarrow{\sigma} M_2$ and $M_2 \gneq M_1$. Let $G := \{s \in S \mid M_1(s) < M_2(s)\}$ and

$$M[G/k] := \begin{cases} k & \text{if } s \in G \\ M(s) & \text{if } s \notin G \end{cases}$$

Prove that for every $k \in \mathbb{N}$, there is a marking M and a transition sequence σ' such that $M_1 \xrightarrow{\sigma'} M$ with $M \geq M_2[G/k]$.

d) **OPTIONAL:** Consider the optimisation where if, when constructing $\mathrm{Cov}(N)$, we find a new extended marking L_2 successor of some $L_1 \in V$, such that there is an L with $M_0 \rightsquigarrow^* L \rightsquigarrow^* L_1$ and $L > L_2$, then we discard L_2 and continue the exploration. Argue why this optimisation is correct.