Concurrency Theory (WS 2016)		Out: Sat, 01 Dec Due: Wed, 07 Dec
	Exercise Sheet 6	
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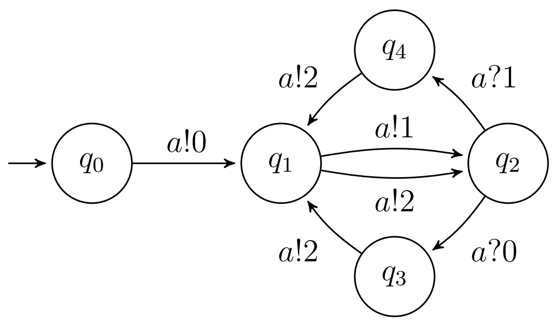
Problem 1: LCS are WSTS

Let $L = (Q, q_0, C, M, \to)$ be a LCS. Recall that configurations are elements of the set $\mathbb{C}_L := Q \times (M^*)^C$ and the initial configuration is $\gamma_0 := (q_0, \boldsymbol{\varepsilon})$ with $\boldsymbol{\varepsilon}(\_) = \varepsilon$. The goal is to prove termination (from γ_0) is decidable.

a) Define a suitable decidable qo $\sqsubseteq \subseteq \mathbb{C}_L \times \mathbb{C}_L$.
[Hint: *Lift the subword wqo on M^* to $\mathbb{C}_L$*]

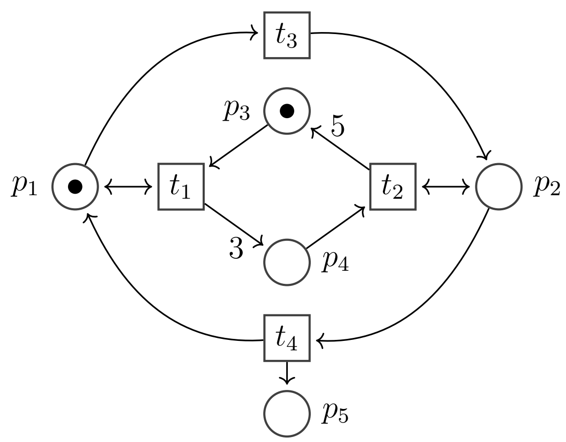
b) Prove that $(\mathbb{C}_L, \to, \sqsubseteq)$ is a (transitive) WSTS.

c) Construct FRT(γ_0) for the following LCS:



Problem 2: Semilinear Forward Inductive Invariants

a) Consider the following Petri net:



Find a semilinear forward inductive invariant and use it to prove that $(1, 0, 3, 2, 0)$ is not reachable.

b) Show how a trap Q gives rise to a semilinear forward inductive invariant.

c) **OPTIONAL** Show that there is a semilinear forward inductive invariant which incorporates the trap property for every trap.

[Hint: *Semilinear sets are closed under intersection*]

Problem 3: Every TS is a WSTS

a) Prove that $(\mathbb{N}_\omega, \leq_\omega)$ is a wqo, where $\mathbb{N}_\omega = \mathbb{N} \uplus \{\omega\}$ and for all $n, n' \in \mathbb{N}$, $n \leq_\omega n'$ if $n \leq n'$ and $x \leq_\omega \omega$ for every $x \in \mathbb{N}_\omega$.

b) Let $T = (S, \rightarrow)$ be a transition system. Define $\ell(s) \in \mathbb{N}_\omega$ for $s \in S$ to be the length of the longest run $s \rightarrow s_1 \rightarrow \cdots$ in T, i.e. a natural if it is finite and ω if it is infinite. Prove that T is well-structured under the order $\leq_\ell$ where $s \leq_\ell s'$ iff $\ell(s) \leq_\omega \ell(s')$.

c) Is $\leq_\ell$ decidable in general?

Problem 4: Simulations and Upward Closed Sets

Let $(S, \rightarrow, \leq_S)$ be a qo transition system. We call a set X *upward closed* if for all $x \in X$ and $s \in S$, if $x \leq s$ then $s \in X$. Equivalently, X is upward closed iff $X = X\uparrow$. Define

$$\mathrm{pre}(X) := \{x \in S \mid x \rightarrow x' \text{ and } x' \in X\}$$

to be the set of predecessors of configurations in X. We also define $\mathrm{pre}^0(X) := X$, $\mathrm{pre}^{n+1}(X) := \mathrm{pre}(\mathrm{pre}^n(X))$, and $\mathrm{pre}^*(I) := \bigcup_{i \in \mathbb{N}} \mathrm{pre}^i(I)$.

Prove the following claim:
$\leq_S$ is a simulation if and only if $\mathrm{pre}^*(I)$ is upward closed for all upward closed sets $I \subseteq S$.