Concurrency theory
Exercise sheet 3

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Out: November 1 Due: November 7

Submit your solutions until Tuesday, November 7, during the lecture. You may submit in groups up to three persons.

Exercise 1: Communication-free Petri nets and SAT

A **communication-free Petri net** (or **BPP net**) is a Petri net in which each transition consumes at most one token, i.e. we have $\forall t \in T\colon \sum_{p \in P} \mathrm{i}(t,p) \in \{0,1\}$.

Show that the coverability problem for communication-free Petri nets is NP-hard by reducing SAT.

To this end, show how to construct in polynomial time from a given Boolean formula φ in conjunctive form φ a communication-free Petri net (N, M_0, M_f) such that M_f is coverable if and only if φ is satisfiable.

Hint: Introduce places for the parts of the formula. A computation of the net should first define a variable assignment, and then evaluate the formula under the assignment.

Remark: In fact, reachability and coverability for communication-free Petri nets are NP-complete.

Exercise 2: 1-safe Petri nets and Boolean programs

Recall that a Petri net (N, M_0) is **1-safe** if we have $M \in \{0,1\}^P$ for all $M \in R(N, M_0)$.

Consider **Boolean programs**, sequences of labeled commands over a fixed number of Boolean variables For simplicity, we restrict ourselves to the following types of commands:

$z \leftarrow x \wedge y$	$z \leftarrow x \vee y$	$z \leftarrow \neg x$
`if` x `then goto` ℓ_t `else goto` ℓ_f	`goto` ℓ	`halt`

Here, x, y, z are variables and ℓ, ℓ_t, ℓ_f are labeles. The semantics of the commands are expected.

Assume that the initial variable assignment is given by $x = \mathit{false}$ for all variables.

Assume that such a Boolean program is given. Explain how to construct an equivalent 1-safe Petri net. Equivalent means that the unique execution of the Boolean program is halting if and only if a certain marking is coverable.

Remark: This proves that coverability for 1-safe Petri nets is PSPACE-hard. In fact, coverability and reachability for 1-safe Petri nets are PSPACE-complete.

Exercise 3: Using a unary encoding

Assume that we measure the size of Petri nets and markings by taking the unary encoding of the numbers, i.e. we redefine $\left|M\right| = \sum_{p \in P}(1 + M(p))$ and $\left|N\right| = \sum_{t \in T, p \in P}(1 + \mathfrak{i}(o, t) + \mathfrak{o}(t, p))$.

a) Does the coverability problem get any easier using this assumption?

   Hint: Inspect the proof of Lipton's result.

b) Discuss whether Rackoff's bound can be improved, proving

$$f(i + 1) \leqslant (n \cdot f(i))^{i+1} + f(i) .$$

Exercise 4: VASS

There are other automata models that are equivalent to Petri nets, but they are less useful to model concurrent systems.

A **vector addition system with states (VASS)** of dimension $d \in \mathbb{N}$ is a tuple $A = (Q, \Delta, q_0, v_0)$ where Q is a finite set of control states, $\Delta \subseteq Q \times \mathbb{Z}^d \times Q$ is a set of transitions, $q_0 \in Q$ is the initial state and $v_0 \in \mathbb{N}^d$ is the initial counter assignment. We write transitions $(q, a, q') \in \Delta$ as $q \xrightarrow{v} q'$. A configuration of a VASS is a tuple (q, v) consisting of a control state $q \in Q$ and a counter assignment, a vector $v \in \mathbb{N}^d$. The initial configuration of interest is (q_0, v_0). A transition (q, a, q') is enabled in some configuration (q'', v) if $q'' = q$ and $(v + a) \in \mathbb{N}^d$ (i.e. $(v + a)_i \geqslant 0$ for all $i \in \left\{1, \ldots, d\right\}$). In this case, it can be fired, leading to the configuration $(q', v + a)$. Reachability is defined as expected.

a) Let (N, M_0, M_f) be a Petri net. Show how to construct a VASS A and a configuration (q_f, v_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

b) Let A be a VASS and (q_f, v_f) a configuration. Show how to construct a Petri net (N, M_0, M_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

c) (Bonus exercise, not graded.) A **vector addition system (VAS)** is a VASS with a single state, i.e. $Q = \left\{q_0\right\}$. Show that VAS-reachability is interreducible with VASS reachability (or Petri net reachability).