Concurrency theory
Exercise sheet 5

TU Braunschweig

Sebastian Muskalla, Prakash Saivasan

Winter term 2017/18

Out: November 16 Due: November 22

Submit your solutions until Wednesday, November 22, during the lecture. You may submit in groups up to three persons.

Exercise 1: Well quasi orderings

1. Prove or disprove that $(\mathbb{N}, /)$ is a well-quasi ordering, here a/b means "a divides b".
2. Let $(A, \leqslant)$ be a WQO. Prove that for any $k \in \mathbb{N}$, $(A^k, \leqslant^k)$ is also a WQO. The ordering $\leqslant^k$ is obtained by component-wise application of $\leqslant$ on the vectors of A^k.

Exercise 2: Multisets are WQO

A (finite) multiset over a set X is a function $m : X \mapsto \mathbb{N}$ such that $\lfloor m \rfloor = \{x \mid x \in X \wedge m(x) > 0\}$ is finite. We denote by $M(X)$ the set of all such multisets. Let $(X, \leqslant_X)$ be a well quasi order and $m_1, m_2 \in M(X)$, an embedding from m_1 to m_2 is defined as an injective function $\phi : \lfloor m_1 \rfloor \mapsto \lfloor m_2 \rfloor$ such that $x \leqslant_X \phi(x)$ and $m_1(x) \leqslant m_2(\phi(x))$ for all $x \in \lfloor m_1 \rfloor$. We say $m_1 \leqslant_M m_2$ if there is an embedding from m_1 to m_2. Prove that $(M(X), \leqslant_M)$ is a WQO.

Exercise 3: Composition of WSTS

Consider two WSTS $TS_1 = (\Gamma_1, \rightarrow_1, \gamma_0, \leqslant_1)$ and $TS_2 = (\Gamma_2, \rightarrow_2, \bar{\gamma}_0, \leqslant_2)$. We define their composition to be $TS_1 \otimes TS_2 = (\Gamma, \rightarrow, \gamma, \leqslant)$ where

- $\Gamma = \Gamma_1 \times \Gamma_2$
- $(\gamma, \bar{\gamma}) \rightarrow (\gamma', \bar{\gamma}')$ iff $\gamma \rightarrow_1 \gamma'$ and $\bar{\gamma} \rightarrow_2 \bar{\gamma}'$
- $\gamma = (\gamma_0, \bar{\gamma}_0)$
- $(\gamma, \bar{\gamma}) \leqslant (\gamma', \bar{\gamma}')$ iff $\gamma \leqslant_1 \gamma'$ and $\bar{\gamma} \leqslant_2 \bar{\gamma}'$

Prove that $TS_1 \otimes TS_2$ is also a WSTS.