Concurrency theory
Exercise sheet 1

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Out: October 24 Due: October 30

Submit your solutions until 18:00 , October 30
You may submit in groups up to three persons.

Exercise 1: Upward-closed sets

For a finite alphabet Σ and $w_1, w_2 \in \Sigma^*$, let $w_1 \leqslant w_2$ if and only if w_1 is a subword of w_2 [i.e. w_1 can be obtained by deleting zero or more letters in w_2]. For any $\mathcal{L} \subseteq \Sigma^*$, the upward-closure of $\mathcal{L}$ is defined as $\mathcal{L}\uparrow = \{w \mid \exists w' \in \mathcal{L} : w' \leqslant w\}$ and the downward closure $\mathcal{L}\downarrow$ is defined as $\mathcal{L}\downarrow = \{w \mid \exists w' \in \mathcal{L} : w \leqslant w'\}$

a) Show that for any language $\mathcal{L} \subseteq \Sigma^*$, the languages $\mathcal{L}\uparrow$ and $\mathcal{L}\downarrow$ are regular. (Assume that the set of finite sequences over a finite alphabet, ordered by the subword relation, is well-quasi-ordered)

b) Let $(A, \leqslant)$ be a wqo and $M_1, M_2 \subseteq A$ finite. Show that it is decidable if $M_1\uparrow = M_2\uparrow$.

Exercise 2: Well quasi orderings

1. Prove or disprove that $(\mathbb{N}, /)$ is a well-quasi ordering, here a/b means "a divides b".

2. Let $(A, \leqslant)$ be a WQO. Prove that for any $k \in \mathbb{N}$, $(A^k, \leqslant^k)$ is also a WQO. The ordering $\leqslant^k$ is obtained by component-wise application of $\leqslant$ on the vectors of A^k.

Exercise 3: Multisets are WQO

A (finite) multiset over a set X is a function $m : X \mapsto \mathbb{N}$ such that $\lfloor m \rfloor = \{x \mid x \in X \wedge m(x) > 0\}$ is finite. We denote by $M(X)$ the set of all such multisets. Let $(X, \leqslant_X)$ be a well quasi order and $m_1, m_2 \in M(X)$, an embedding from m_1 to m_2 is defined as an injective function $\phi : \lfloor m_1 \rfloor \mapsto \lfloor m_2 \rfloor$ such that $x \leqslant_X \phi(x)$ and $m_1(x) \leqslant m_2(\phi(x))$ for all $x \in \lfloor m_1 \rfloor$. We say $m_1 \leqslant_M m_2$ if there is an embedding from m_1 to m_2. Prove that $(M(X), \leqslant_M)$ is a WQO.