Concurrency theory
Exercise sheet 9

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Winter term 2018/19

Out: January 18 Due: January 23

Submit your solutions until Wednesday, January 23, 12:00 am. You may submit in groups up to three persons.

Exercise 1: Trace robustness strictly implies reachability robustness
Prove the following Lemma from the lecture.

a) If $\mathrm{Tr}_{\mathrm{TSO}}(P) = \mathrm{Tr}_{\mathrm{SC}}(P)$ for some program, then $\mathrm{Reach}_{\mathrm{TSO}}(P) = \mathrm{Reach}_{\mathrm{SC}}(P)$.

Here, $\mathrm{Reach}_{\mathrm{TSO}}(P) = \{pc \mid cf_0 \rightarrow^*_{\mathrm{TSO}} (pc, val, buf) \text{ with } buf(i) = \varepsilon \text{ for all } i\}$ and $\mathrm{Reach}_{\mathrm{SC}}(P)$ is obtained by restricting the definition to computations in which each issue (STORE) is followed by the store (UPDATE).

b) The reverse implication does not hold.

Remark: Relations
Recall the following basic definitions for **relations**.

Let N be a set and let $\leqslant \subseteq N \times N$ be a relation.

Recall that N is **reflexive** if $x \leqslant x$ for all $x \in N$. It is **antisymmetric** if $x \leqslant y$ and $y \leqslant x$ imply $x = y$ (for all $x, y \in N$). It is **transitive** if $x \leqslant y$ and $y \leqslant z$ imply $x \leqslant z$ (for all $x, y, z \in N$). If all three properties hold, we call $\leqslant$ a **partial order**.

A partial order is called **total** (or linear) if any two elements are comparable, i.e.
$\forall x, y \in N \colon x \leqslant y$ or $y \leqslant x$.

We let $\leqslant^*$ denote the reflexive-transitive closure of $\leqslant$, the smallest subset of $N \times N$ that contains $\leqslant$ and is reflexive and transitive.

We may see $(N, \leqslant)$ as a directed graph. We call $\leqslant$ **acyclic** if this graph does not contain a non-trivial cycle $x_0 \leqslant x_1 \leqslant \ldots \leqslant x_m \leqslant x_0$. (Cycles of the shape $x_0 \leqslant x_0$ are trivial.)

Exercise 2: Relations
Let N be a **finite** set and let $\leqslant \subseteq N \times N$ be a relation.

a) Explain how to construct $\leqslant^*$ from $\leqslant$ within a finite number of steps.

b) Prove that $\leqslant^*$ is a partial order (i.e. antisymmetric) if and only if $\leqslant$ is acyclic.

c) Now assume that $\leqslant_{po}$ is some partial order. Prove that there is a total order $\leqslant_{to} \subseteq N \times N$ containing $\leqslant_{po}$, i.e. $\leqslant_{po} \subseteq \leqslant_{to}$.

Exercise 3: Shasha and Snir

Prove the Lemma by Shasha and Snir:

A trace $\mathrm{Tr}(\tau) \in \mathrm{Tr}_{\mathrm{TSO}}(P)$ is in $\mathrm{Tr}_{\mathrm{SC}}(P)$ if and only if its happens-before relation $\rightarrow_{\mathrm{hb}}$ is acyclic.

Proceed as follows:

a) Show that for traces of SC computations, $\rightarrow_{\mathrm{hb}}$ is necessarily acyclic.

b) Show how from a trace with acyclic $\rightarrow_{\mathrm{hb}}$, one can construct an SC computation τ' with $\mathrm{Tr}(\tau') = \mathrm{Tr}(\tau)$.

Hint: Use Exercise 2.