Concurrency theory
Exercise sheet 2

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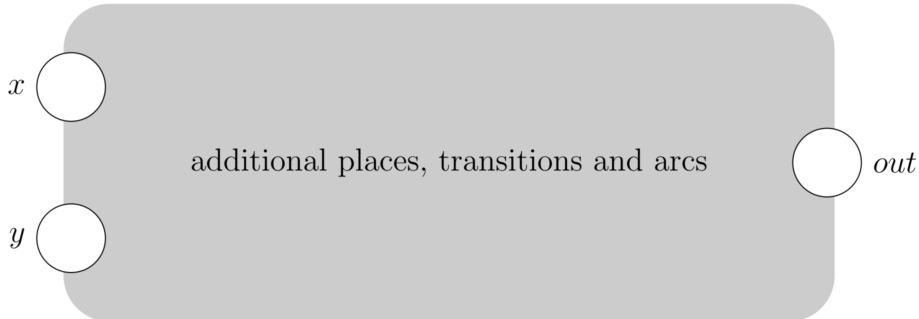
Out: November 06 Due: November 12

Submit your solutions until Tuesday, November 12, during the lecture. You may submit in groups up to three persons.

Exercise 1: Addition and multiplication

Consider the (incomplete) Petri net containing places x, y and *out* depicted below.



a) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) = m + n$.

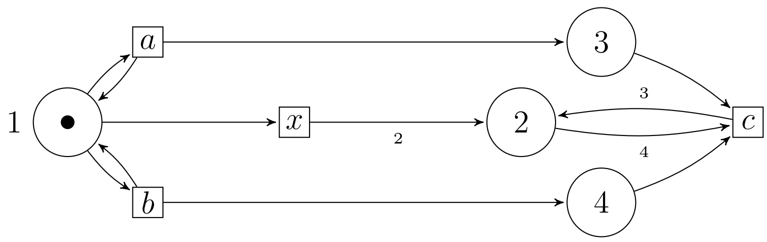
(*Terminating* means that no transition is enabled anymore.)

b) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) \in \{0, \ldots, m \cdot n\}$.

In each part of this exercise, argue briefly that your construction is correct.

Exercise 2: Rackoff's bound

Consider the Petri net $N = (\{1, 2, 3, 4\}, \{a, b, c, x\}, \text{in}, \text{out})$ with multiplicites as depicted below. The initial marking of interest is $M_0 = (1, 0, 0, 0)^T$ and the final marking is $M_f = (1, 0, 10, 100)^T$.



Compute the values $m(3, M_0)$ and $f(3)$ and argue why they are correct.

Exercise 3: Communication-free Petri nets and SAT

A **communication-free Petri net** (or **BPP net**) is a Petri net in which each transition consumes at most one token, i.e. we have $\forall t \in T\colon \sum_{p \in P} \mathrm{in}(t, p) \in \{0, 1\}$.

Show that the coverability problem for communication-free Petri nets is NP-hard by reducingi from 3-SAT.

To this end, show how to construct in polynomial time from a given Boolean formula φ in conjunctive normal form a communication-free Petri net (N, M_0, M_f) such that M_f is coverable if and only if φ is satisfiable.

Hint: Introduce places for the parts of the formula. A computation of the net should first define a variable assignment, and then evaluate the formula under the assignment.

Remark: In fact, reachability and coverability for communication-free Petri nets are NP-complete.