Concurrency theory
Exercise sheet 2

TU Braunschweig

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Out: November 13 Due: November 19

Submit your solutions until Tuesday, November 19, during the lecture. You may submit in groups up to three persons.

Exercise 1: 1-safe Petri nets and Boolean programs

Recall that a Petri net (N, M_0) is **1-safe** if we have $M \in \{0,1\}^P$ for all $M \in R(N, M_0)$.

Consider **Boolean programs**, sequences of labeled commands over a fixed number of Boolean variables. For simplicity, we restrict ourselves to the following types of commands:

$z \leftarrow x \wedge y$	$z \leftarrow x \vee y$	$z \leftarrow \neg x$
`if` x `then goto` ℓ_t `else goto` ℓ_f	`goto` ℓ	`halt`

Here, x, y, z are variables and ℓ, ℓ_t, ℓ_f are labeles. The semantics of the commands are expected.

Assume that the initial variable assignment is given by $x = \textit{false}$ for all variables x.

Assume that a Boolean program is given. Explain how to construct an equivalent 1-safe Petri net. Equivalent means that the unique execution of the Boolean program is halting if and only if a certain marking is coverable.

Remark: This proves that coverability for 1-safe Petri nets is PSPACE-hard. In fact, coverability and reachability for 1-safe Petri nets are PSPACE-complete.

Exercise 2: Using a unary encoding

Assume that we measure the size of Petri nets and markings by taking the unary encoding of the numbers, i.e. we redefine $|M| = \sum_{p \in P}(1 + M(p))$ and $|N| = \sum_{t \in T, p \in P}(1 + \mathfrak{i}(o,t) + \mathfrak{o}(t,p))$.

a) Does the coverability problem get any easier using this assumption?

Hint: Inspect the proof of Lipton's result.

b) Discuss whether Rackoff's bound can be improved, proving

$$f(i+1) \leqslant (n \cdot f(i))^{i+1} + f(i) \ .$$

Exercise 3: VASS

There are other automata models that are equivalent to Petri nets, but they are less useful to model concurrent systems.

A **vector addition system with states (VASS)** of dimension $d \in \mathbb{N}$ is a tuple $A = (Q, \Delta, q_0, v_0)$ where Q is a finite set of control states, $\Delta \subseteq Q \times \mathbb{Z}^d \times Q$ is a set of transitions, $q_0 \in Q$ is the initial state and $v_0 \in \mathbb{N}^d$ is the initial counter assignment. We write transitions $(q, a, q') \in \Delta$ as $q \xrightarrow{a} q'$. A configuration of a VASS is a tuple (q, v) consisting of a control state $q \in Q$ and a counter assignment, a vector $v \in \mathbb{N}^d$. The initial configuration of interest is (q_0, v_0). A transition (q, a, q') is enabled in some configuration (q'', v) if $q'' = q$ and $(v+a) \in \mathbb{N}^d$ (i.e. $(v+a)_i \geqslant 0$ for all $i \in \{1, \ldots, d\}$). In this case, it can be fired, leading to the configuration $(q', v + a)$. Reachability is defined as expected.

a) Let (N, M_0, M_f) be a Petri net. Show how to construct a VASS A and a configuration (q_f, v_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

b) Let A be a VASS and (q_f, v_f) a configuration. Show how to construct a Petri net (N, M_0, M_f) such that (q_f, v_f) is reachable from (q_0, v_0) in A if and only if M_f is reachable from M_0 in N.

c) (Bonus exercise, not graded.) A **vector addition system (VAS)** is a VASS with a single state, i.e. $Q = \{q_0\}$. Show that VAS-reachability is interreducible with VASS reachability (or Petri net reachability).