Exercises to the lecture
Concurrency Theory
Sheet 9

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Peter Chini Delivery until 04.02.2020 at 12:00

Exercise 9.1 (Lossy Channel Systems with conditionals)
We extend lossy channel systems with a transition that, given a channel c and a word $w \in \Sigma^*$ checks whether w is contained in c as a subword:

$$q \xrightarrow{\text{check } w \text{ in } c} p$$

Formally, we extend the transition relation on configurations of a lossy channel systems by the rule:

$$(q, W) \to (p, W) \text{ if } q \xrightarrow{\text{check } w \text{ in } c} p \text{ and } w \leq W(c).$$

Given an extended lossy channel system $L = (Q, q_0, C, M, \to_L)$, construct a lossy channel system $L' = (Q', q_0, C, M', \to_{L'})$ with $Q \subseteq Q'$ and $M \subseteq M'$ such that:

$$(q_1, W) \to_L^* (q_2, W') \text{ if and only if } (q_1, W) \to_{L'}^* (q_2, W'),$$

for each $q_1, q_2 \in Q$ and W, W' channel contents.

Exercise 9.2 (Ideals)
Let $(C, \leq)$ be a well quasi ordering. An *ideal* is a subset $I \subseteq C$ which is non-empty, downward closed, and directed. Directed means that for each $x, y \in I$ there exists a $z \in I$ such that $x \leq z$ and $y \leq z$.

a) Let $(A, \leq_A)$ and $(B, \leq_B)$ be two wqos and $(A \times B, \leq)$ their product. Show that a subset $J \subseteq A \times B$ is an ideal if and only if $J = I_A \times I_B$ where I_A, I_B are ideals in A and B, respectively.

   Hint: For one direction you need to show that $J = \mathit{proj}_A(J) \times \mathit{proj}_B(J)$, where proj_A denotes the projection to A: $\mathit{proj}_A(a, b) = a$.

b) Show that the ideals of $(\mathbb{N}, \leq)$ are $\mathbb{N}$ itself and all sets of the form $n\downarrow$ for $n \in \mathbb{N}$.

c) Derive that the ideals of $\mathbb{N}^d$ are of the form $M\downarrow$, where M is a generalized marking, a vector $M \in (\mathbb{N} \cup \{\omega\})^d$.

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