Games with perfect information

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Prof. Dr. Roland Meyer — Summer term 2017

Out: May 11 — Due: May 19

Submit your solutions until Friday, May 19, 14:00, in the box next to office 343.

Exercise 1

Let $\trianglelefteq$ be a partial order on some set V.

a) Let $X, Y \subseteq V$ be subsets of V with $X \subseteq Y$. Prove that $X\uparrow \subseteq Y\uparrow$.

   Does $\mathrm{Min}(X) \subseteq \mathrm{Min}(Y)$ also hold?

b) Prove that the union of upward-closed sets is again upward closed.

c) Prove Lemma 6.8 from the lecture notes:
   Let $X \subseteq V$ be upward closed, i.e. $X = X\uparrow$, then $X = \mathrm{Min}(X)\uparrow$.

   Hint: Prove both inclusions separately. For one inclusion, you can use Part a).

Exercise 2: The subword relation

Let Σ be some fixed, finite, non-empty alphabet. We consider the set of words Σ^* over Σ.

We define the **subword relation** $\preceq$ on Σ^* as follows: We have $v \preceq w$ if v can be obtained from w by deleting letters. This means that $w = a_0 a_1 \dots a_k$ for some $a_i \in \Sigma$, and $v = a_{j_0} a_{j_1} \dots a_{j_\ell}$ for $0 \leq j_0 < j_1 < \dots < j_\ell \leq k$.

For example, consider the alphabet $\{a, b\}$ and $w = aba$. The words $\varepsilon, a, b, aa, ab, ba, aba$ are smaller with respect to $\preceq$ than w.

a) Prove that $\preceq$ is a partial order.

b) For each of the following languages over $\Sigma = \{a, b, c\}$, each represented by a regular expression, present their minimal elements and check whether they are upward-closed.

   - $a\Sigma^* b\Sigma^* c$
   - $ab \cup b\Sigma^* a \cup aabb$
   - $c\Sigma^+ c$

   Recall that $\Sigma^+ = \Sigma^* \setminus \{\varepsilon\}$.

c) Let $w \in \Sigma$ be a word. How can one obtain a representation of the upward closure of the singleton set containing w, i.e. $\{w\}\uparrow$?

Exercise 3: A not so intricate scheduling problem

Consider the instance of MOFST with the tasks $\mathcal{T} = \{\tau_1, \tau_2, \tau_3\}$ specified by the table below, and $m = 2$ processors.

	C_τ	D_τ	T_τ
τ_1	1	1	2
τ_2	2	2	2
τ_3	1	2	2

Construct and solve the scheduling game for this input.