Games with perfect information

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Exercise sheet 12

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Out: June 30 Due: July 7

Submit your solutions until Friday, July 7, 14:00, in the box next to office 343.

Exercise 1

Formally prove Lemma 10.14 from the lecture notes:

Let p be a finite play of $\mathcal{G}^{inf}$. Then $\text{purge}^*(p)$ is a valid play of $\mathcal{G}^{fin}$ ending the same position.

Conclude the statement of Lemma 10.16 from the lecture notes:

If $s^{fin}_{\star}$ is a positional strategy, then so is the lifted strategy $s^{inf}_{\star}$.

Exercise 2: Parallel-free and bipartite

a) Throughout the whole lecture, we have assumed that the game arena is parallel-free, meaning there is at most one arc from some position to another.

   Assume you are given a game arena that is not parallel free. Show how to construct an equivalent game arena that is parallel-free

   - for reachability/Büchi/parity games,
   - for mean payoff games.

b) In the section on mean payoff games, we have assumed that the game arena is bipartite and the players alternately take turns.

   Assume you are given a non-bipartite game arena. Show how to construct an equivalent bipartite game arena for reachability/Büchi/parity games.

   Does this also work for mean payoff games?

Here, by equivalent game arena, we mean that for each player, there is a map $\iota_{\star}: V_{\star} \to V'_{\star}$ embedding her positions in the old game arena into the set of positions of the new game arena. Furthermore, we want that a position $x \in V$ is winning for player $\star$ if and only if $\iota(x)$ is winning for $\star$.

Exercise 3: Büchi games as mean payoff games

Let G be a finite, bipartite, deadlock-free game arena, and let B be the winning set for a Büchi game from a fixed initial position x_0.

Show how this Büchi game can be transformed into a mean payoff game.

Assume the initial position x_0 is winning for some player $\star$ in the Büchi game. How is this reflected in the mean payoff game? Make your argumentation formal!