Modern SAT Solvers

Albert Schimpf

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Propositional Satisfiability Problem (SAT)

SAT

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- First NP-complete problem

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- Recent development enabled industry use

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Focus: conflict-driven SAT solvers

Specifically: depth-first search and backtrack based

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Improvements:

Describe high level structure

Deduction phase

Variable selection

Data structure

Backtracking

1 Basic State Transition System

2 Non-Chronological Backtracking

3 Two-Watched-Literals Scheme

4 VSIDS Heuristic

5 Summary: The Modern Solver

Basic Definitions

Main idea: describe solving process precisely.

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- Formula F
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- Conflict management set C
- Transition Rules (e.g. unit, decision etc.)

Basic Definitions - Trail M

- Recursive calls: Introduce checkpoint symbol $\diamondsuit$

red: variable assignment already visited

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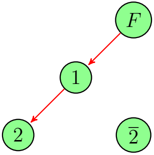

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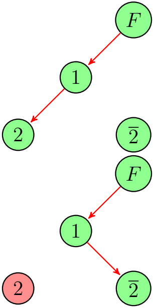

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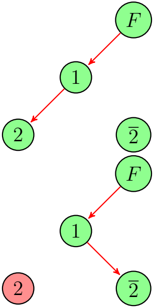

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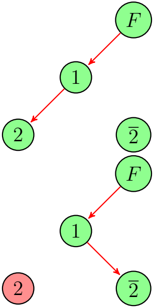

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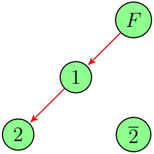

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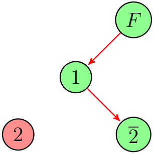

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Checkpoints allow for iterative implementation.

Transition Rules: Decide

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Transition Rules: Conflict and Backtrack

$$(Conflict)$$

$$C = no\_cflct$$

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$$\frac{}{C = no\_cflct \text{ and } \overline{l_1} \vee ... \vee \overline{l_k} \in F} (Conflict)$$

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$$(Backtrack)$$

$C = \{l, l_1, ..., l_k\}$

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$C = \{l, l_1, ..., l_k\}$ & $\bar{l} \vee \overline{l_1} \vee ... \vee \overline{l_k} \in F$

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(((Conflict; Backtrack) || UnitPropag)* ; [Decide])*

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$$< F, [\Diamond 12 \Diamond 34], no\_cflct > \xrightarrow{Decide} < F, [\Diamond 12 \Diamond 34 \Diamond 5], no\_cflct >$$

Classic DPLL Run

$$F_{init} = \{\overline{1} \vee 2, \overline{3} \vee 4, \overline{5} \vee 6, \overline{2} \vee \overline{5} \vee \overline{6}\}$$

$$\begin{array}{cccc}
< F, [], no_cflct > & \xrightarrow{Decide} & < F, [\Diamond 1], no_cflct > & \xrightarrow{UnitPropag} \\
< F, [\Diamond 12], no_cflct > & \xrightarrow{Decide} & < F, [\Diamond 12 \Diamond 3], no_cflct > & \xrightarrow{UnitPropag} \\
< F, [\Diamond 12 \Diamond 34], no_cflct > & \xrightarrow{Decide} & < F, [\Diamond 12 \Diamond 34 \Diamond 5], no_cflct > & \xrightarrow{UnitPropag}
\end{array}$$

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$$\begin{array}{cccc}
< F, [], no_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 1], no_cflct > & \xrightarrow{UnitPropag} \\
< F, [\diamondsuit 12], no_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 12 \diamondsuit 3], no_cflct > & \xrightarrow{UnitPropag} \\
< F, [\diamondsuit 12 \diamondsuit 34], no_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 5], no_cflct > & \xrightarrow{UnitPropag} \\
< F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 56], no_cflct > & & &
\end{array}$$

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$$\begin{array}{|cccc|} \hline < F, [], no\_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 1], no\_cflct > & \xrightarrow{UnitPropag} \\ < F, [\diamondsuit 12], no\_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 12 \diamondsuit 3], no\_cflct > & \xrightarrow{UnitPropag} \\ < F, [\diamondsuit 12 \diamondsuit 34], no\_cflct > & \xrightarrow{Decide} & < F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 5], no\_cflct > & \xrightarrow{UnitPropag} \\ < F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 56], no\_cflct > & \xrightarrow{Conflict} & & \end{array}$$

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< F, [\Diamond 12 \Diamond 34 \Diamond 56], no_cflct > & \xrightarrow{Conflict} & < F, [\Diamond 12 \Diamond 34 \Diamond 56], \{2, 5, 6\} > &
\end{array}$$

Classic DPLL Run

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$< F, [], no\_cflct >$	$\xrightarrow{Decide}$	$< F, [\Diamond 1], no\_cflct >$	$\xrightarrow{UnitPropag}$
$< F, [\Diamond 12], no\_cflct >$	$\xrightarrow{Decide}$	$< F, [\Diamond 12 \Diamond 3], no\_cflct >$	$\xrightarrow{UnitPropag}$
$< F, [\Diamond 12 \Diamond 34], no\_cflct >$	$\xrightarrow{Decide}$	$< F, [\Diamond 12 \Diamond 34 \Diamond 5], no\_cflct >$	$\xrightarrow{UnitPropag}$
$< F, [\Diamond 12 \Diamond 34 \Diamond 56], no\_cflct >$	$\xrightarrow{Conflict}$	$< F, [\Diamond 12 \Diamond 34 \Diamond 56], \{2, 5, 6\} >$	$\xrightarrow{Backtrack}$

Classic DPLL Run

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< F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 56], no_cflct > & \xrightarrow{\underline{Conflict}} & < F, [\diamondsuit 12 \diamondsuit 34 \diamondsuit 56], \{2, 5, 6\} > & \xrightarrow{\underline{Backtrack}} \\
< F, [\diamondsuit 12 \diamondsuit 34\overline{5}], no_cflct > & & &
\end{array}$$

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\end{array}}$$

1. Basic State Transition System
2. Non-Chronological Backtracking
3. Two-Watched-Literals Scheme
4. VSIDS Heuristic
5. Summary: The Modern Solver

Motivation

$$(\neg x_1 \vee x_2) \wedge (\neg x_3 \vee x_4) \wedge (\neg x_5 \vee \neg x_6) \wedge (x_6 \vee \neg x_5 \vee \neg x_2)$$

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Decision	Unit propagation
x1 = 1	x2 = 1
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$(\neg x_1$

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Transform trail into an *implication graph*:

Motivation

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Reason for the conflict: Decision 1 and 5, or $(\overline{1} \vee \overline{5})$

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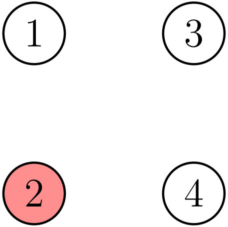

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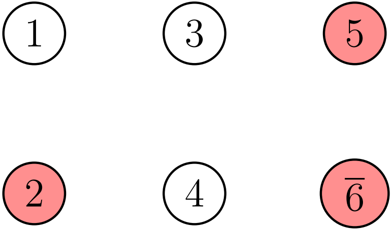

Reason for the conflict: Decision 1 and 5, or $(\overline{1} \vee \overline{5})$

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$$M = \{\Diamond x_1 x_2 \Diamond x_3 x_4 \Diamond x_5 \overline{x}_6\}$$

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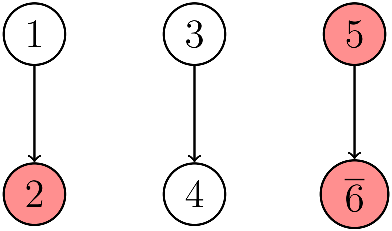

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Goals

- Determine reason of current conflict

Goals

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- Jump over decision levels that are irrelevant

Goals

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- Jump over decision levels that are irrelevant
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Constructing Partial Implication Graphs

Consider a state $< F, M, \{\overline{1}, 2, \overline{3}\} >$ where F = $(\overline{9} \vee \overline{6} \vee 7 \vee \overline{8}) \wedge (8 \vee 7 \vee \overline{5}) \wedge (\overline{6} \vee 8 \vee 4) \wedge (\overline{4} \vee \overline{1}) \wedge (\overline{4} \vee 5 \vee 2) \wedge (5 \vee 7 \vee \overline{3}) \wedge (1 \vee \overline{2} \vee 3\} \cup F_{other}$ and

$M = \{...6...\overline{7}...\diamondsuit 9\overline{8}\overline{5}4\overline{1}2\overline{3}\}$

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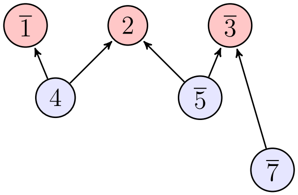
Work backwards starting with conflict literals



Constructing Partial Implication Graphs

Consider a state $< F, M, \{\overline{1}, 2, \overline{3}\} >$ where F = $(\overline{9} \lor \overline{6} \lor 7 \lor \overline{8}) \land (8 \lor 7 \lor \overline{5}) \land (\overline{6} \lor 8 \lor 4) \land (\overline{4} \lor \overline{1}) \land (\overline{4} \lor 5 \lor 2) \land (5 \lor 7 \lor \overline{3}) \land (1 \lor \overline{2} \lor 3\} \cup F_{other}$ and

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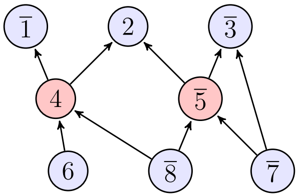
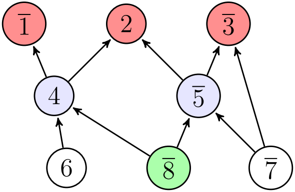
Stop at first unique implication point (UIP)



Unique Implication Point and Backjump Clause

$$M = \{\ldots 6 \ldots \overline{7} \ldots \diamondsuit 9 \overline{8} \overline{5} 4 \overline{1} 2 \overline{3}\}$$



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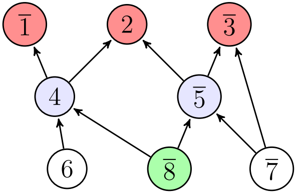

- Stop at *first* UIP ($\overline{8}$)

Unique Implication Point and Backjump Clause

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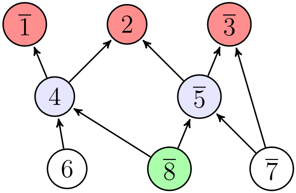
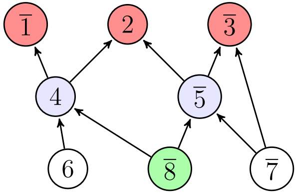


- Stop at *first* UIP ($\overline{8}$)
- Reason of the conflict are all literals without incoming edges

Unique Implication Point and Backjump Clause

$$M = \{\ldots 6 \ldots \overline{7} \ldots \Diamond 9 \overline{8} \overline{5} 4 \overline{1} 2 \overline{3}\}$$



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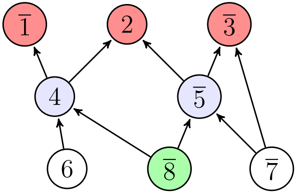

- Stop at *first* UIP ($\overline{8}$)
- Reason of the conflict are all literals without incoming edges
- Backjump clause is the negated disjunction of reason literals ($\overline{6} \vee 8 \vee 7$)
- Jump to *second most recent decision level* of backjump clause

Resolution

Apply resolution until first UIP is found.
In this case: stop resolution if only one literal is in current decision level. Previous example:

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$$\frac{5 \vee 7 \vee \mathbf{\overline{3}} \qquad\qquad 1 \vee \overline{2} \vee \mathbf{3}}{}$$

Resolution step can now be defined as a rule and integrated in the state transition system.

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$$\dfrac{\bar{4} \vee 5 \vee \mathbf{2} \qquad \dfrac{5 \vee 7 \vee \bar{\mathbf{3}} \qquad 1 \vee \bar{2} \vee \mathbf{3}}{5 \vee 7 \vee 1 \vee \bar{\mathbf{2}}}}{}$$

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$$\cfrac{\bar{4} \lor \bar{\mathbf{1}} \qquad \cfrac{\bar{4} \lor 5 \lor \mathbf{2} \qquad \cfrac{5 \lor 7 \lor \bar{\mathbf{3}} \qquad 1 \lor \bar{2} \lor \mathbf{3}}{5 \lor 7 \lor 1 \lor \bar{\mathbf{2}}}}{\bar{4} \lor 5 \lor 7 \lor \mathbf{1}}}{}$$

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$$\dfrac{\bar{6} \vee 8 \vee \mathbf{4} \qquad \dfrac{\bar{4} \vee \bar{\mathbf{1}} \qquad \dfrac{\bar{4} \vee 5 \vee \mathbf{2} \qquad \dfrac{5 \vee 7 \vee \bar{\mathbf{3}} \qquad 1 \vee \bar{2} \vee \mathbf{3}}{5 \vee 7 \vee 1 \vee \bar{\mathbf{2}}}}{\bar{4} \vee 5 \vee 7 \vee \mathbf{1}}}{5 \vee 7 \vee \bar{\mathbf{4}}}}{}$$

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Apply resolution until first UIP is found.
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$$M = \{...6...\overline{7}...\diamondsuit 9\overline{8}54\overline{1}2\overline{3}\}$$

$$\dfrac{8 \vee 7 \vee \mathbf{\overline{5}} \qquad \dfrac{\overline{6} \vee 8 \vee \mathbf{4} \qquad \dfrac{\overline{4} \vee \mathbf{\overline{1}} \qquad \dfrac{\overline{4} \vee 5 \vee \mathbf{2} \qquad \dfrac{5 \vee 7 \vee \mathbf{\overline{3}} \qquad 1 \vee \overline{2} \vee \mathbf{3}}{5 \vee 7 \vee 1 \vee \mathbf{\overline{2}}}}{\overline{4} \vee 5 \vee 7 \vee \mathbf{1}}}{5 \vee 7 \vee \mathbf{\overline{4}}}}{\overline{6} \vee 8 \vee 7 \vee \mathbf{5}}}{}$$

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$$\frac{8 \lor 7 \lor \overline{\mathbf{5}} \qquad \dfrac{\overline{6} \lor 8 \lor \mathbf{4} \qquad \dfrac{\overline{4} \lor \overline{\mathbf{1}} \qquad \dfrac{\overline{4} \lor 5 \lor \mathbf{2} \qquad \dfrac{5 \lor 7 \lor \overline{\mathbf{3}} \qquad 1 \lor \overline{2} \lor \mathbf{3}}{5 \lor 7 \lor 1 \lor \overline{\mathbf{2}}}}{\overline{4} \lor 5 \lor 7 \lor \mathbf{1}}}{5 \lor 7 \lor \overline{\mathbf{4}}}}{\overline{6} \lor 8 \lor 7 \lor \mathbf{5}}}{8 \lor 7 \lor \overline{6}}$$

Resolution step can now be defined as a rule and integrated in the state transition system.

Theorem - Correctness

Theorem (Correctness): All runs of DPLL are finite. If, initialized with the set of clauses F_{init}, DPLL terminates in the state $\langle F, M, C \rangle$, then: (a) C = no_cflct or C = $\emptyset$; (b) If C = $\emptyset$ then F_{init} is unsatisfiable; (c) If C = no_cflct, then M is a model for F_{init}.

Experiments

Instance	CB	NCB
bf0432-079	>3000	3,73
ssa2670-141	>3000	101,69
ii16e1	>3000	0,53
par16-1-c	165,75	1362,53
flat200-39	656,55	1472,53
4blocksb	>3000	639,87
logistics.c	>3000	38,96
barrel5	189,88	635,12
queueinvar16	>3000	22,9
dlx2_aa	>3000	32,35
2dlx...	>3000	>3000
cnf-r3-b4-k1.2	>3000	18,69

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Basic Definitions

- Formula F is stored as:

Basic Definitions

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- Clauses need to know their current status

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- Lose ability to implement techniques which require whole clause status
- Most used deduction technique in modern solvers: unit propagation

Two-Watched-Literals Scheme

Main idea: optimize unit clause detection

- Non-lazy: Variables store pointers to all clauses with literals of that variable

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- Lazy: Reduce pointer count to a minimum to determine unit status
- Status of only *2 literals per clause* is needed to determine unit status

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- Backtracking has **no** computational cost
- Allows for solving larger instances than before

Experiments

Instance	NCBcb	NCBwl
bf0432-079	3,73	3,11
ssa2670-141	101,69	22,91
ii16e1	0,53	0,48
par16-1-c	1362,53	233,53
flat200-39	1472,53	396,95
sw100-49	17,15	12,19
4blocksb	639,87	248,04
logistics.c	38,96	27,39
facts7hh.13.simple	8,31	9,09
barrel5	635,12	146,74
queueinvar16	22,9	13,06
dlx2_aa	32,35	11,74
dlx2_cc_a_bug17	4,2	3,92
2dlx...	>3000	>3000
cnf-r3-b4-k1.2	18,69	16,48

- Overall big improvement

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- Overall big improvement
- Not enough for hardest instances

1. Basic State Transition System
2. Non-Chronological Backtracking
3. Two-Watched-Literals Scheme
4. VSIDS Heuristic
5. Summary: The Modern Solver

Motivation

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- Mostly useful for random instances
- Do not capture relevant information about structured problems

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Experiments

Instance	NCBwIRSTslis	NCBwIRSTvsids
bf0432-079	3,17	2,24
ssa2670-141	23,22	1,06
ii16e1	0,5	0,42
par16-1-c	198,51	178,36
flat200-39	125,95	12,96
sw100-49	10,3	2,35
4blocksb	232,74	85,94
logistics.c	30,99	26,8
facts7hh.13.simple	9,91	7,28
barrel5	175,74	41,07
queueinvar16	13,36	16,39
dlx2_aa	12,31	10,12
dlx2_cc_a_bug17	3,58	2,86
2dlx_cc_mc_ex_bp_f2_bug005	197,55	41,66
cnf-r3-b4-k1.2	16,78	15,39

- Big improvement over static heuristic

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- Big improvement over static heuristic
- Problem are now solved in acceptable time

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We are now able to describe the modern Solver *Chaff*:

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The state transition introduced helps to implement the solver and guarantees correctness.