SS 2012 2. Mai 2012

Exercises to the lecture Logics
Sheet 2

Jun.-Prof. Dr. Roland Meyer Due 15. Mai 2012 12:00 Uhr

Exercise 2.1 [Recursive decidability]
A set M is called *recursively decidable* if there is an algorithm that, given w as input, halts after a finite amount of time and

- outputs „1“ if $w \in M$ and
- outputs „0“ if $w \notin M$.

A set M is called *recursively enumerable*, if there is an algorithm that outputs a (possibly infinite) sequence in which each element occurs if and only if it is contained in M.

Let Ax and R be recursively decidable sets of formulae in the calculus $\mathcal{F} = \mathcal{F}(\mathrm{Ax}, R)$. Show that

a) the set of proofs in the calculus (Ax, R) is decidable and

b) $T(\mathcal{F})$ is recursively enumerable.

Here, a short description of the involved algorithms (e.g. using bullet points) is sufficient.

Note: $\mathcal{F}$ is not necessarily the same as $\mathcal{F}_0$.

Exercise 2.2 [Inconsistency]
Show that $\Sigma \vdash_{\mathcal{F}_0} A$ if and only if $\Sigma \cup \{\neg A\}$ is inconsistent.

Exercise 2.3 [Proving in $\mathcal{F}_0$]
Prove the following statements without relying on the completeness of $\mathcal{F}_0$. You can, however, use Exercise 2.2, the deduction theorem (for $\mathcal{F}_0$) and the theorems from “Beispiel 1.22” on the slides.

a) $\vdash_{\mathcal{F}_0} (A \to \neg A) \to \neg A$.

b) If B is an axiom in $\mathcal{F}_0$, then $\vdash_{\mathcal{F}_0} (A \to \neg B) \to \neg A$.

c) $\vdash_{\mathcal{F}_0} \neg(A \to B) \to (B \to A)$.

Exercise 2.4 [Complete sets of connectives]
Using structural induction, show that $\{\to, \neg\}$ is a complete set of connectives.

Delivery: until 15. Mai 2012 12:00 Uhr into the box next to room 34/401.4