SS 2012 13.6.2012

Exercises to the lecture Logics
Sheet 5

Jun.-Prof. Dr. Roland Meyer Due 26.6.2012 12:00 Uhr

**Exercise 5.1** [Cardinality of domains I]
For each interpretation $I = (D, I_c, I_v)$, let $|I|$ be defined as $|D|$.

a) Let $n$ be a natural number. Present a formula $A$ in predicate logic involving "=" such that for every interpretation $I$, we have $I \vDash A$ if and only if $|I| \geqslant n$.

b) Present a formula with the above property that, in addition, does not contain the predicate "=".

**Exercise 5.2** [Cardinality of domains II]
We say an interpretation $I = (D, I_c, I_v)$ is *finite* if the set $D$ is finite.

a) Present a formula $A$ such that $I \vDash A$ if and only if $|I| = 1$.

b) Let $B$ be a formula without the predicate symbol "=". Given a finite interpretation $I$ with $I \vDash B$, how can you construct an interpretation $I'$ with $|I'| = |I| + 1$ and $I' \vDash B$? A proof for $I' \vDash B$ is not absolutely necessary here.

c) Deduce from b) that there is no formula without "=" that is equivalent to the formula $A$ above.

**Exercise 5.3** [A satisfiability check]

a) Present an algorithm that, given a formula $A$ and a finite interpretation $I$, decides whether $I \vDash A$. *Note:* This means you have shown that satisfaction under a given finite interpretation is decidable.

b) Let $A$ be a formula of the form $\exists x_1 \cdots \exists x_n B$, in which $B$ contains no quantifier. Show: If $A$ is satisfiable, then it has a model $I$ with $|I| \leqslant n + |B|$. (We say that $A$ exhibits a *small model property*.)

c) Prove using a) and b): Given a formula $A \equiv \exists x_1 \cdots \exists x_n B$ as above, it can be decided algorithmically whether $A$ is satisfiable.

**Exercise 5.4** [Modelling]

a) Describe function symbols and predicate symbols that model name, address, and preferred political party of individuals (e.g. in a database). It should be possible that for certain individuals, not all the data is available. In particular, you should specify the arity and the intended meaning of the function and predicate symbols.

b) Formalize the following integrity constraint: "If a person prefers party $\mathsf{P}$ or $\mathsf{L}$, then their name and address are available."

**Delivery: until 26.6.2012 12:00 Uhr into the box next to room 34/401.4**