Exercises to the lecture Logics
Sheet 6

Jun.-Prof. Dr. Roland Meyer   Due 10.07.2012 12:00 Uhr

Exercise 6.1 [Non-complete theories]

Consider the axiom system Ax that contains the first four axioms from "Beispiel 3.12" on slide 169.

a) Present a model of Ax in which the addition is not commutative.

b) Conclude from a) that the theory generated by Ax is not complete.

Exercise 6.2 [Derivations in $\mathcal{F}$ and $\mathcal{F}_0$]

Let A' be a formula in (first order) predicate logic. Furthermore, assume that A' is obtained from A by replacing atomic formulae p_i by formulae B_i, where A is a formula in propositional logic. Moreover, suppose $\vdash_{\mathcal{F}_0} A$. Show that then $\vdash_{\mathcal{F}} A'$.

Exercise 6.3 [Completeness and Consistency]

Show that a theory T is complete if and only if there is no formula A such that $\text{Th}(T \cup \{A\})$ and $\text{Th}(T \cup \{\neg A\})$ are inconsistent. *Note:* Hence, you have shown that completeness means that the theory cannot be extended consistently in two ways that contradict each other.

Exercise 6.4 [Decidable Theories]

Let $\mathcal{R}$ be a structure such that $T_{\mathcal{R}}$ can be axiomatized by a recursively enumerable set of formulae. Show that then $T_{\mathcal{R}}$ is recursively decidable. (The terms *recursively decidable* and *recursively enumerable* have been defined on sheet 2.)

Delivery: until 10.07.2012 12:00 Uhr into the box next to room 34/401.4