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In-class Exercises to the Lecture Logics  
Sheet 3

Jun.-Prof. Dr. Roland Meyer

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**Exercise 3.1** [König's Lemma]

*Smullyan's ball game* is played by one player according to the following rules: There is a container that has capacity for an unlimited number of balls. Furthermore, there is an unlimited supply of balls, each of which is labeled with a natural number. Initially, there is one ball in the container. In each step, the player can remove a ball from the container and put in an arbitrary number of balls, provided that each of the new balls has a label smaller than the removed ball. The game is over when no further step is possible, i.e. when the container is empty.

Show that each game is over after a finite number of steps.

**Exercise 3.2** [The subsumption rule for the Davis-Putnam method]

Let  $F = \{K_1, \dots, K_n\}$  be a formula in KNF in set notation. Moreover, let  $K_i \subseteq K_j$ . Show that  $F \models F'$ , in which  $F' = F \setminus \{K_j\}$ .

**Exercise 3.3** [The DNF and tableaux]

Devise a method that, using tableaux, produces a DNF for a given formula in propositional logic.