SS 2013 May 29, 2013

Exercises to the lecture Logics
Sheet 4

Jun.-Prof. Dr. Roland Meyer Due June 7, 2013, 12:00pm

Exercise 4.1 [Resolution Calculus]

a) Let K_1, K_2 be clauses and I be a literal with $I \in K_1$ and $\neg I \in K_2$. Moreover, let R be obtained by resolving K_1 and K_2 with respect to I. Show that $\{K_1, K_2\} \models R$.
b) Using the Resolution Calculus, show that $(((p \to q) \wedge (q \to r)) \to \neg(\neg r \wedge p))$ is a tautology.

Exercise 4.2 [Resolution as a fixed point computation]
For a set A of clauses, we write

$$\mathrm{Res}(A) = A \cup \{K \mid K \text{ is a resolvent of two clauses in } A\}.$$

$$\mathrm{Res}^0(A) = A, \quad \mathrm{Res}^{i+1}(A) = \mathrm{Res}(\mathrm{Res}^i(A)) \text{ for } i \geqslant 1.$$

a) Formulate the resolution calculus as a stepwise computation of the sets $\mathrm{Res}^i(A)$.
b) Formulate a theorem stating soundness and (refutation) completeness of your algorithm.
c) Show that your algorithm terminates.

Exercise 4.3 [Dual Formulae]

a) For each valuation φ, let φ' defined by $\varphi'(p) = 1 - \varphi(p)$ for each variable p. Show that for any formula A, we have $\varphi'(d(A)) = 1 - \varphi(A)$.
b) Deduce from a) that for any formula A, the following holds: A is a tautology if and only if $d(A)$ is unsatisfiable. Note that in each direction, every valuation has to be treated.

Exercise 4.4 [Negation Normal Form]
Using structural induction, prove that for any formula there is an equivalent formula in negation normal form. *Hint:* In order to make the induction work, show by induction that A as well as $\neg A$ has a negation normal form.

Exercise 4.5 [Tableaux]
Let Σ be a set of formulas and p, q be atomic formulae with $\Sigma \vdash_\tau p$ and $\Sigma \vdash_\tau p \to q$. Prove that then $\Sigma \vdash_\tau q$.

Delivery: until June 7, 2013, 12:00pm into the box next to room 34/401.4