SS 2013 24. April 2013

In-class Exercises to the Lecture Logics
Sheet 1

Jun.-Prof. Dr. Roland Meyer Discussion on 25./26. April 2013

Exercise 1.1 [Compactness Theorem]

Prove the corollary of the Compactness Theorem: For a set Σ of formulae and a formula A, we have $\Sigma \models A$ if and only if there is a finite subset $\Sigma_0 \subseteq \Sigma$ with $\Sigma_0 \models A$.

Exercise 1.2 [Compactness Theorem]

Let $\Sigma_0 \subseteq \Sigma_1 \subseteq \Sigma_2 \subseteq \cdots$ be a sequence of satisfiable sets of formulae. Show that $\Sigma = \bigcup_{i \geqslant 0} \Sigma_i$ is satisfiable as well.

Exercise 1.3 [Normal forms]

A *literal* is a formula of the form p or $\neg p$, where p is an atomic formula. A *clause* or *dual clause* is a formula of the form

$$L_1 \vee \cdots \vee L_n \quad \text{or} \quad L_1 \wedge \cdots \wedge L_n,$$

respectively, where $L_1, \ldots, L_n$ are literals. A formula is in *conjunctive normal form (CNF)* or *disjunctive normal form (DNF)*, if it is of the form

$$K_1 \wedge \cdots \wedge K_n \quad \text{or} \quad K_1 \vee \cdots \vee K_n$$

where $K_1, \ldots, K_n$ are clauses or dual clauses, respectively. Shortly:

- a formula in CNF is a conjunction of disjunctions of literals.
- a formula in DNF is a disjunction of conjunctions of literals.

Devise a method that, given a truth table, constructs a formula in

a) DNF

b) CNF

with the given truth table.

Exercise 1.4 [Satisfiability checks]

The *satisfiability problem* asks for a given formula whether it is satisfiable or not. It is generally conjectured that there is no efficient method to solve this problem.

a) Present an efficient method to decide whether a given formula in *DNF* is satisfiable or not.

b) In Exercise 1.3 you saw that it is possible to construct an equivalent formula in DNF for any given formula. Explain why the following is not an efficient algorithm for the satisfiability problem:

1. Calculate an equivalent formula in DNF.
2. Use the efficient method from a) to check for satisfiability.

Exercise 1.5 [Compactness and tiling problems]

Suppose you want to tile $\mathbb{Z} \times \mathbb{Z}$. You are given a finite set of *tile types*, each of which specifies the colors of the four sides. You want to tile the plane in a way so that neighboring colors match.

Prove: There is a tiling of the whole plane if and only if every finite section can be tiled.